For every observable there is an operator.

Observable		Operator
position x	$\longleftrightarrow$	x
momentum p	$\longleftrightarrow$	$-i\hbar \frac{d}{dx}$
energy E	$\longleftrightarrow$	$i\hbar \frac{d}{dt}$

$$E \longleftrightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

In general, each measurement of observable $\mathcal{O}$ can give different results O_n.

The importance of eigenstates

$$\hat{O}\,\phi_n(x,t) = o_n\,\phi_n(x,t)$$

observable

eigenvalue

o_n - each eigenvalue is a possible value we can find

IF $\Psi(x,t) = \phi_n(x,t)$,

we will only get o_n.

IF $\Psi = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2)$, what can we get? o_1 and o_2

Momentum

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$-i\hbar \frac{\partial}{\partial x} e^{ikx} = \hbar k e^{ikx}$$

e^{ikx} is an eigenstate of momentum.

Ground state in a box

$$\Psi_1(x,t) = \sqrt{\frac{2}{a}} \sin(k_1 x) e^{-i\omega_1 t}$$

$$k_1 = \frac{\pi}{a}, \quad \omega_1 = \frac{\hbar k_1^2}{2m}$$

$$\sin(k_1 x) = \frac{1}{2i}\left[e^{ik_1 x} - e^{-ik_1 x}\right]$$

$\Rightarrow$ measure momentum : k_1 or $-k_1$

Expectation value

= average value

$$\langle \hat{O} \rangle = \int \Psi^*(x,t)\, O\, \Psi(x,t)\, dx$$

$$\langle \hat{p} \rangle = \int \Psi^*(x,t) \left[-i\hbar \frac{d}{dx}\right] \Psi(x,t)\, dx$$

$$\langle \hat{x} \rangle = \int \Psi^*(x,t)\, [x]\, \Psi(x,t)\, dx$$

$$\langle E \rangle = \int \Psi^*(x,t) \left[i\hbar \frac{d}{dt}\right] \Psi(x,t)\, dx$$

$$\langle \hat{x}^2 \rangle = \int \Psi^*(x,t)\, [x^2]\, \Psi(x,t)\, dx$$

$$\langle \hat{p}^2 \rangle = \int \Psi^*(x,t) \left[-\hbar^2 \frac{d^2}{dx^2}\right] \Psi(x,t)\, dx$$

Ground state $\Psi = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) e^{-i\omega_1 t}$

$$\langle x \rangle = \int_0^a \Psi^* \, x \, \Psi \, dx$$

$$= \frac{2}{a} \int_0^a \sin\left(\frac{\pi x}{a}\right) e^{i\omega_1 t} \; x \cdot \sin\left(\frac{\pi x}{a}\right) e^{-i\omega_1 t} \, dx$$

$$= \frac{2}{a} \int_0^a x \sin^2\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{a}{2}$$

{ We will look at 'tricks' for doing these integrals later.

$$\langle p\rangle = \int_0^a \Psi^* \left[-i\hbar\frac{d}{dx}\right]\Psi$$

$$= -i\hbar\left(\frac{2}{a}\right)\int_0^a \sin\left(\frac{\pi x}{a}\right)\left[\frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right)\right]dx$$

$$= 0$$

$$\langle p^2\rangle = \int_0^a \Psi^* \left[-\hbar^2\frac{d^2}{dx^2}\right]\Psi$$

$$= -\hbar^2\left(\frac{2}{a}\right)\int_0^a \sin\left(\frac{\pi x}{a}\right)\left[\frac{-\pi^2}{a^2}\sin\left(\frac{\pi x}{a}\right)\right]$$

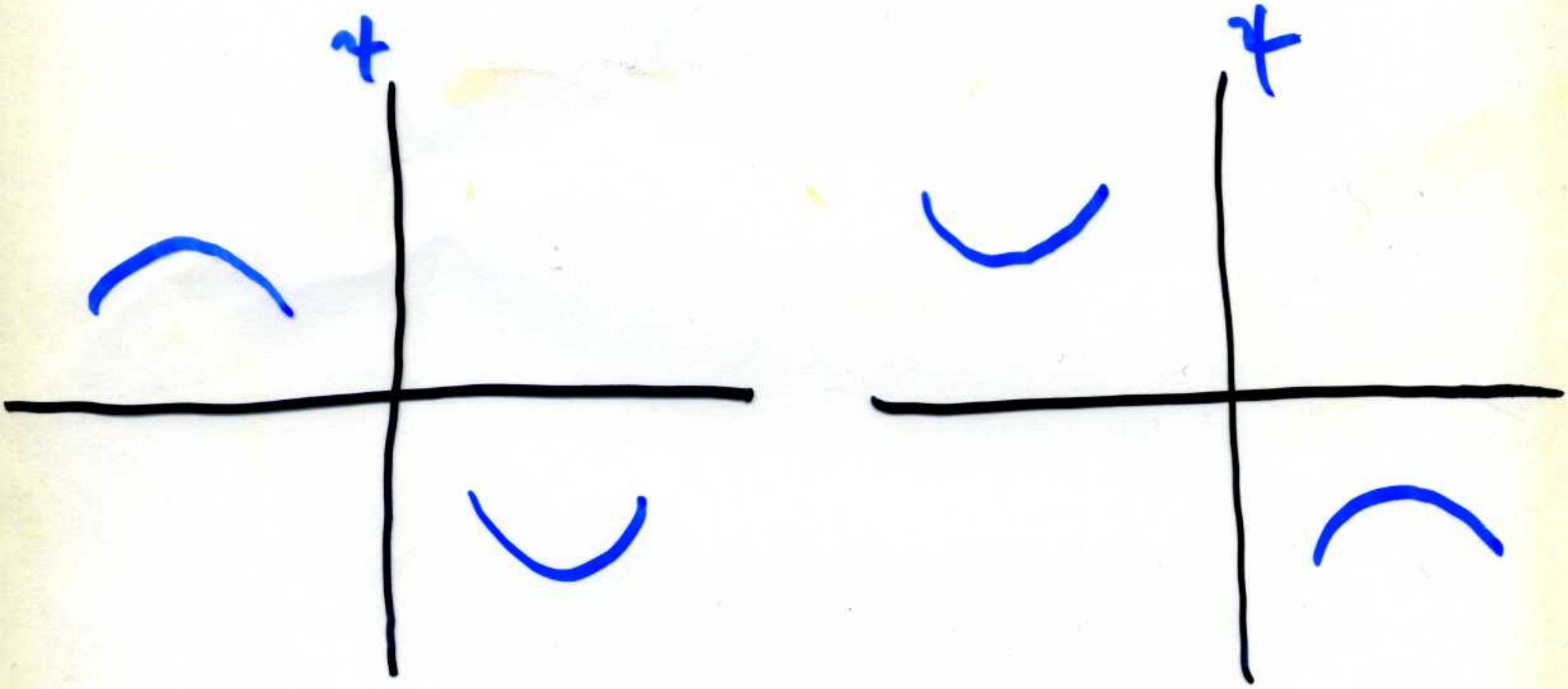
$$= \frac{\hbar^2\pi^2}{a^2}$$

Momentum is either $\frac{\hbar\pi}{a}$ or $-\frac{\hbar\pi}{a}$, so ...

$$-\frac{d^2}{dx^2}\psi_E(x) = \frac{2m}{\hbar^2}\left[E - V(x)\right]\psi_E(x)$$

$E - V > 0$ 'allowed'

$E - V < 0$ 'Forbidden'



$E - V > 0$

$\Downarrow$

curves back towards $\psi = 0$

$E - V < 0$

$\Downarrow$

curves away from $\psi = 0$