Probability

$\hat{O}|\phi_n\rangle = o_n|\phi_n\rangle$

State at time t $|\psi(t)\rangle$

Measure $\hat{O}$:

Possible results: o_n

probability of o_n: $|\langle\phi_n|\psi(t)\rangle|^2$

Expectation value = average

$$\langle\hat{O}\rangle = \langle\psi(t)|\hat{O}|\psi(t)\rangle$$

$$= \langle\psi(t)|\hat{O}\left\{\sum_n |\phi_n\rangle\langle\phi_n|\right\}|\psi(t)\rangle$$

$$= \sum_n \underbrace{|\langle\phi_n|\psi(t)\rangle|^2}_{\text{Prob of } o_n} \underbrace{o_n}_{\times\ o_n}$$

Dirac delta function $\delta(x)$

$$\int_{-\infty}^{\infty} dx\ \delta(x) f(x) = f(0)$$

$$\int_{-\infty}^{\infty} dx\ \delta(x-x_0) f(x) = f(x_0)$$

$$\langle x | x' \rangle = \delta(x'-x)$$

- note this is ϕ (zero) unless $x' = x$

$$\langle p | p' \rangle = \int dx\ \langle p | x \rangle \langle x | p' \rangle$$

$$= \int dx\ \frac{e^{-ipx}}{\sqrt{2\pi}} \frac{e^{ip'x}}{\sqrt{2\pi}}$$

$$= \int \frac{dx}{2\pi} e^{i(p'-p)x} = \delta(p'-p)$$

What happens after a measurement?

Measure $\hat{O}$, $\hat{O}|\phi_n\rangle = o_n|\phi_n\rangle$

If result at time t_0 is o_n, state 'collapses' to $|\phi_n\rangle$ at time t_0.

After t_0, state evolves according to Schrödinger equation:

$t > t_0$: $|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\phi_n\rangle$