Physics 632 } WI06 } Sample First Midterm Questions

1) $H = \frac{L^2}{2I} + \mu B L_x$

(a) What is the ground state if $\mu B = \frac{1}{I}$?

(b) For $\mu B = \frac{1}{I}$, at time $t=0$ the state is:

$$|\psi(0)\rangle = |1,1\rangle,$$

where $L^2|lm\rangle = l(l+1)|lm\rangle$ and $L_z|lm\rangle = m|lm\rangle$.

What is $\langle L_z\rangle(t) = \langle\psi(t)|L_z|\psi(t)\rangle$?

Interpret your result.

Note: $L_x|l,m_x\rangle = m_x|l,m_x\rangle$

$$|1,1_x\rangle = \frac{1}{2}\{|1,-1\rangle + \sqrt{2}|1,0\rangle + |1,1\rangle\}$$

$$|1,0_x\rangle = \frac{1}{\sqrt{2}}\{|1,1\rangle - |1,-1\rangle\}$$

$$|1,-1_x\rangle = \frac{1}{\sqrt{2}}\{|1,-1\rangle - \sqrt{2}|1,0\rangle + |1,1\rangle\}$$

$$|1,1\rangle = \frac{1}{2}\{|1,-1_x\rangle + \sqrt{2}|1,0_x\rangle + |1,1_x\rangle\}$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\{|1,1_x\rangle - |1,-1_x\rangle\}$$

$$|1,-1\rangle = \frac{1}{2}\{|1,-1_x\rangle - \sqrt{2}|1,0_x\rangle + |1,1_x\rangle\}$$

$$\hbar c = \frac{1}{2\pi}\left(12345 \text{ eV-Å}\right)$$

3) A hydrogen atom at time $t=0$ is in a mixed state:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left\{|100\rangle + |200\rangle\right\},$$

where $|nlm\rangle$ are eigenstates of H with energy eigenvalues $E_n = -\frac{1}{n^2}R$ where $R \simeq +13.6$ eV.

The reduced mass is about $\mu c^2 \simeq 5.11 \times 10^5$ eV.

$$\langle\vec{r}|\psi(0)\rangle = \psi(\vec{r}, t=0) = \frac{1}{\sqrt{2}}\left\{\langle\vec{r}|100\rangle + \langle\vec{r}|200\rangle\right\}$$

$$\langle\vec{r}|100\rangle = \frac{1}{\pi^{1/2}a_0^{3/2}}e^{-r/a_0}$$

$$\langle\vec{r}|200\rangle = \frac{1}{\pi^{1/2}(2a_0)^{3/2}}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0} \qquad a_0 = 0.53 \text{ Å}$$

(a) What is $\psi(\vec{r},t)$?

(b) What is the probability per unit length of finding an electron at distance r from the proton?

$$P(r)\, r^2 dr = \int d\Omega \left|\psi^*(\vec{r},t)\,\psi(\vec{r},t)\right| r^2 dr$$

What is $P(r)$?

2) A particle is confined to a sphere of radius R, so

$$H = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} + V(r), \qquad V(r) = \begin{cases} 0 & r<R \\ \infty & r>R \end{cases}$$

$\boxed{\text{eigenstates}}$ $\psi_{n\ell m}(\vec{r}) = N_{\ell n}\, j_\ell(k_{\ell n} r)\, Y_\ell^m(\theta,\phi)$ for $r<R$

$\psi(\vec{r}) = 0$ for $r>R$

$\boxed{\text{boundary condition}}$ $j_\ell(k_{\ell n} R) = 0$ $\quad r=R$

$\boxed{\text{norm}}$ $\int d^3r\, \psi^+(\vec{r})\psi(\vec{r}) = N_{\ell n}^2 \int_0^\infty r^2 dr\, \left|j_\ell(k_{\ell n} r)\right|^2 = 1$

energy: $E_{\ell n} = \dfrac{k_{\ell n}^2}{2m}$

At $t=0$, $\psi(\vec{r}, t=0) = \sqrt{\dfrac{6}{\pi R^3}}\; \Theta\left(r - \frac{R}{2}\right)$,

where $\Theta(r-\frac{R}{2}) = 1$ for $r<\frac{R}{2}$ and $\Theta(r-\frac{R}{2}) = 0$ for $r>\frac{R}{2}$.

(a) What is $\psi(\vec{r},t)$? Show that you only need $j_0(k_{0n} r) = \dfrac{\sin(k_{0n} r)}{k_{0n} r}$, find N_{0n} and k_{0n}. Evaluate all integrals. Leave your final result as a sum.

(b) What is $\langle r\rangle(t)$? Interpret your result.