Physics 632 { WI06 } Notes for 1st midterm

$\boxed{\hbar = c = 1}$

Angular momentum

$$L^2 |l, m\rangle = l(l+1)|lm\rangle \qquad L_z|lm\rangle = m|lm\rangle$$

$$L_\pm = L_x \pm i L_y \; ; \quad L_\pm |l,m\rangle = \sqrt{l(l+1) - m(m\pm 1)}\, |l, m\pm 1\rangle$$

$$L^2 |l, m_x\rangle = l(l+1)|l, m_x\rangle \; , \quad L_x |l, m_x\rangle = m_x |l, m_x\rangle$$

$$|1, 1_x\rangle = \frac{1}{2}\left\{ |1,-1\rangle + \sqrt{2}\,|1,0\rangle + |1,1\rangle \right\}$$

$[m, l]$

$$|1, 0_x\rangle = \frac{1}{\sqrt{2}}\left\{ |1,1\rangle - |1,-1\rangle \right\}$$

$$|1, -1_x\rangle = \frac{1}{2}\left\{ |1,-1\rangle - \sqrt{2}\,|1,0\rangle + |1,1\rangle \right\}$$

$$|1, 1\rangle = \frac{1}{2}\left\{ |1,-1_x\rangle + \sqrt{2}\,|1,0_x\rangle + |1,1_x\rangle \right\}$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}\left\{ |1,1_x\rangle - |1,-1_x\rangle \right\}$$

$$|1, -1\rangle = \frac{1}{2}\left\{ |1,-1_x\rangle - \sqrt{2}\,|1,0_x\rangle + |1,1_x\rangle \right\}$$

Particle in a Sphere

$$H = \frac{P^2}{2m} + V(r) \; , \quad V(r) = \begin{cases} 0 & r < R \\ \infty & r > R \end{cases}$$

$$\langle \vec{r}\,|nlm\rangle = \phi_{nlm}(\vec{r}) = N_{ln}\, j_l(k_{ln} r)\, Y_l^m(\theta, \phi)$$

for $l = m = 0$:
$$\phi_{n00}(\vec{r}) = \sqrt{\frac{2}{R}}\, k_{0n}\, j_0(k_{0n} r)\, Y_0^0(\theta, \phi)$$

$$j_0(x) = \frac{\sin(x)}{x} \; , \quad j_1(x) = \frac{\sin(x)}{x^2} - \frac{\cos(x)}{x}$$

$$j_l(k_{ln} R) = 0 \;\Rightarrow\; \text{for } l = 0 \quad \sin(k_{0n} R) = 0$$

norm:
$$N_{ln}^2 \int_0^R r^2 dr\, |j_l(k_{ln} r)|^2 = 1$$

Spherical Harmonics $\quad \int d\Omega$ is $\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta$

orthonormality $\quad \int d\Omega \, Y_\ell^{m*}(\theta,\phi) \, Y_{\ell'}^{m'}(\theta,\phi) = \delta_{\ell\ell'} \delta_{mm'}$

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \; ; \quad Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{i\phi} \, , \quad Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_1^{-1} = \sqrt{\frac{3}{8\pi}} \sin\theta \, e^{-i\phi}$$

Hydrogen $\quad H = \frac{p^2}{2\mu} - \frac{\alpha}{r} \; , \quad \alpha = \frac{1}{137}$

Bohr radius $\quad a_0 = \frac{1}{\alpha\mu} \; , \quad \mu \simeq 0.511 \text{ MeV}$

Rydberg constant $\quad \mathcal{R} = \frac{1}{2\mu a_0^2} \simeq 13.6 \text{ eV}$

eigenstates $|n\ell m\rangle$ have $E_n = -\frac{\mathcal{R}}{n^2}$

$$\langle \vec{r} \, | \, n\ell m \rangle = \phi_{n\ell m}(\vec{r}) \quad [\text{new } \phi\text{'s}]$$

$$\phi_{100} = \frac{2}{a_0^{3/2}} e^{-r/a_0} \, Y_0^0$$

$$\phi_{200} = \frac{2}{(2a_0)^{3/2}} \left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0} \, Y_0^0$$

$$\begin{pmatrix} \phi_{211} \\ \phi_{210} \\ \phi_{21-1} \end{pmatrix} = \frac{1}{\sqrt{3}\,(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \begin{pmatrix} Y_1^1 \\ Y_1^0 \\ Y_1^{-1} \end{pmatrix}$$

Integrals

$k \neq p$ $$\int dx \, \sin(kx)\sin(px) = \frac{\sin[(k-p)x]}{2(k-p)} - \frac{\sin[(k+p)x]}{2(k+p)}$$

$k \neq p$ $$\int dx \, \sin(kx)\cos(px) = -\frac{\cos[(k-p)x]}{2(k-p)} - \frac{\cos[(k+p)x]}{2(k+p)}$$

$k \neq p$ $$\int dx \, \cos(kx)\cos(px) = \frac{\sin[(k-p)x]}{2(k-p)} + \frac{\sin[(k+p)x]}{2(k+p)}$$

$$\int dx \, x \sin(kx) = -\frac{x}{k}\cos(kx) + \frac{1}{k^2}\sin(kx)$$

$$\int dx \, x^2 \sin(kx) = -\frac{(-2+k^2x^2)}{k^3}\cos(kx) + \frac{2x}{k^2}\sin(kx)$$

$k \neq p$ $$\int dx \, x \sin(kx)\sin(px) = \frac{1}{2}\left\{\frac{\cos[(k-p)x]}{(k-p)^2} - \frac{\cos[(k+p)x]}{(k+p)^2} + \frac{x\sin[(k-p)x]}{k-p} - \frac{x\sin[(k+p)x]}{k+p}\right.$$

$$\int dx \, x \sin^2(kx) = -\frac{\cos(2kx) - 2k^2x^2 + 2kx\sin(2kx)}{8k^2}$$

$$\int_0^\infty dr \, r^n e^{-br} = \frac{n!}{b^{n+1}}$$