PHYSICS 632 Winter 2006 FINAL

Write your name on your test booklet.

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) A particle with spin 1 ($s_1 = 1$) interacts with a particle with spin 1/2 ($s_2 = 1/2$). The hamiltonian is:

$$H = \lambda \, \mathbf{S_1} \cdot \mathbf{S_2}$$

(a) Give a complete set of eigenstates (show all quantum numbers and say what each is in words; e.g., s is the total spin) and the energy eigenvalue for each state.

(b) At time $t = 0$ the z-component of the first particle's spin is measured to be 0 and the z-component of the second particle's spin is measured to be 1/2. At a later time t_1, the z-component of the first particle's spin is measured again. What are the possible results and their respective probabilities?

2) A particle is confined to an infinite one-dimensional square well so that the wave function is non-zero only for $0 < x < a$. A perturbing potential, $V(x) = -\alpha \; \delta(x - a/2)$, is added. (a) Compute the first-order shift in all energies. (b) Compute the second-order shift in the ground state energy, leaving your answer as a sum but clearly stating or showing which terms are zero (if any). Is this second-order shift finite or infinite?

3) Two non-identical spin-1/2 particles ($s_1 = 1/2$, $s_2 = 1/2$) are fixed in space, so that we only need their spin state. To leading order:

$$H_0 = \omega \, \mathbf{S_1} \cdot \mathbf{S_2},$$

where $\omega > 0$. (a) What are the eigenstates and eigenvalues? (b) A perturbing potential, $V = \lambda S_{1z}$, is added. Note that only the first particle is involved in V. Compute the first-order perturbative shifts of the energies for each eigenstate. (c) Compute the second-order shift in the energy of the ground state.