Physics 632 WI 05 Sample midterm 2 questions.

1) $M = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

a) What are M^T and M^+ ?

b) What are the eigenstates and eigenvalues of M ?

c) What is $M^+ M$?

d) What is M^{-1} ?

2) A hyrogen atom is in the state :

$$\psi(\vec{r},0) = \frac{4}{(2a_0)^{3/2}}\left[\frac{e^{-r/a_0}}{\sqrt{4\pi}} + A\frac{r}{a_0}e^{-r/2a_0}\left(iY_1^1 - Y_1^0\right)\right]$$

a) What are the possible measurements of L^2 and what is the probability for each at time t ?

b) What are the possible measurements of L_z and the probabilities for each ?

c) At $t = t_0$, L_z is measured and gives 0. What is the expectation value of $D_z = q\hat{z}$ for times $t > t_0$?

3) A spin-$\frac{1}{2}$ particle has two eigenstates $|+\rangle$ and $|-\rangle$, where $J^2|\pm\rangle = \frac{3}{4}\hbar^2|\pm\rangle$ and $J_z|\pm\rangle = \pm\frac{1}{2}\hbar|\pm\rangle$. At $t=0$ J_y is measured and $\frac{1}{2}\hbar$ is found. What is $|\psi(t)\rangle$ if $H = \omega_0 J_z$? What are the possible measurements of J_x at later times and the probability for each?

4) Two spin-$\frac{1}{2}$ particles have eigenstates $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, where $J_1^2|\psi\rangle = J_2^2|\psi\rangle = \frac{3}{4}\hbar^2|\psi\rangle$ for all, $J_{1z}|\pm, m_2\rangle = \pm\frac{1}{2}\hbar|\pm m_2\rangle$, $J_{2z}|m_1, \pm\rangle = \pm\frac{1}{2}\hbar|m_1, \pm\rangle$, etc. $H = \frac{J^2}{2I} + \omega_0 J_z$, where $\vec{J} = \vec{J}_1 + \vec{J}_2$. At $t=0$ J_{1z} and J_{2z} are measured and J_{1z} gives $\frac{1}{2}\hbar$ while J_{2z} gives $-\frac{1}{2}\hbar$. What is $|\psi(t)\rangle$?

5) A hydrogen atom is placed in a magnetic field so that $H = H_{coulomb} + \omega_0 L_z$ governs the relative motion, with $H_{coulomb} = \frac{P_r^2}{2\mu} + \frac{L^2}{2\mu r^2} - \frac{e^2}{r}$.

$$|\psi(0)\rangle = \frac{1}{\sqrt{3}}|211\rangle - \sqrt{\frac{2}{3}}|21-1\rangle$$

What is $|\psi(t)\rangle$? If L_x is measured at time t, what are the possible results and associated probabilities?

6) Rotations about the z-axis are given by:

$$\vec{r}\,' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \hat{R}(\phi_1)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\phi_1 & \sin\phi_1 \\ -\sin\phi_1 & \cos\phi_1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

a) What are $\hat{R}^{-1}(\phi_1)$ and $\hat{R}^{+}(\phi_1)$?

b) If $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$ what is $\langle\psi|\psi\rangle$? What is ψ'? What is $\langle\psi'|\psi'\rangle$?

c) Show that $\hat{R}(\phi_2)\hat{R}(\phi_1) = \hat{R}(\phi_1+\phi_2)$.