PHYSICS 632 **Winter 2005** **Mar. 10 final**

Name: Solutions

30 pts.

1) Two particles are in a one-dimensional well of width a. (a) If the particles are identical bosons with no "spin", what are the energies of the ground state and first excited state and what is the degeneracy of each of these states? What is the first excited state wave-function written using the one-body particle-in-a-well eigenstates? If there is more than one possible first excited state, pick one and tell me which one you are picking. (b) Same problem for two identical fermions with no spin.

(a) $\epsilon_n = n^2 \epsilon_1$, $\epsilon_1 = \frac{\hbar^2 \pi^2}{a^2}$ – these are 1-body energies

two bosons: $E_{gd.} = 2\epsilon_1$ – not degenate, only one possible state

$E_{1st\,excited} = \epsilon_1 + 4\epsilon_1 = 5\epsilon_1$ - not degenearate

$$\psi_{1st}(x_1, x_2, t) = \frac{e^{-i5\epsilon_1 t/\hbar}}{\sqrt{2}} \left\{ \phi_1(x_1)\phi_2(x_2) + \phi_1(x_2)\phi_2(x_1) \right\}$$

(b) two Fermions - use exclusion principle

$E_{gd.} = \epsilon_1 + \epsilon_2 = 5\epsilon_1$ – not degenerate

$E_{1st} = \epsilon_1 + \epsilon_3 = 10\epsilon_1$ – not degenerate

$$\psi_{1st}(x_1, x_2, t) = \frac{e^{-i10\epsilon_1 t/\hbar}}{\sqrt{2}} \left\{ \phi_1(x_1)\phi_3(x_2) - \phi_1(x_2)\phi_3(x_1) \right\}$$

40 pts.

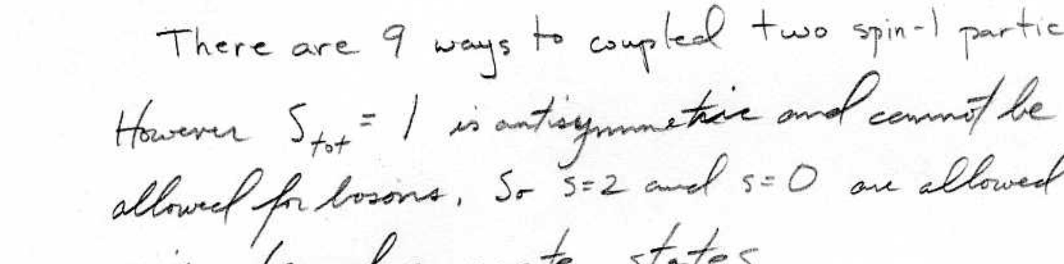
2) Two spin-1 bosons are confined to a sphere of radius a. (a) What is the ground state wave function $\psi(\mathbf{r_1}, \mathbf{r_2}, s_1, s_2)$? Show the radial, angular and spin parts of the wave-function, and since there are many possiblities you must pick one. What is the degeneracy of the ground state? (b) If the interaction $\delta H = b\mathbf{S_1} \cdot \mathbf{S_2}$, with b positive, is added, the ground state level is split into many levels. What are the energies of these levels and what quantum numbers label them?

(a) $\psi(\vec{r}_1, \vec{r}_2, s_1, s_2) = A_{10}^2 R_{10}(r_1) R_{10}(r_2) Y_0^0(\hat{r}_1) Y_0^0(\hat{r}_2) \, |\underbrace{2}_{S_{tot}}\, \underbrace{2}_{m_s}\rangle$

There are 9 ways to coupled two spin-1 particles, However $S_{tot} = 1$ is antisymmetric and cannot be allowed for bosons, So $s=2$ and $s=0$ are allowed and give $\underline{\underline{6}}$ degenerate states.

(b) $b\vec{S}_1 \cdot \vec{S}_2 = \frac{b}{2}(S^2 - S_1^2 - S_2^2) = \frac{b}{2}(S^2 - 4\hbar^2)$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$. Only $s = 0$ and 2 are allowed, so we get



labelled by $n=0, l=0, s_1=1, s_2=1, s, m_s$

$s = 0$ or 2, $m_s = -s, -s+1 \ldots s$

note that $s=1$ is not allowed because it is antisymmetric

30 pts.

3) Four electrons are confined to a sphere of radius a. In addition to the confining interaction the interaction $\delta H = b\mathbf{L}\cdot\mathbf{S}$ is added. (a) If $b > 0$ what is the ground state? Give the shell configuration and a complete set of allowed quantum numbers of your choice to define the ground state. (b) Same as (a) for $b < 0$.

shell configuration throughout $(1s)^2(1p)^2$

The interaction $\vec{L}\cdot\vec{S}$ is zero for the $(1s)^2$ electrons, so we focus on the $(1p)^2$ electrons.

$b\vec{L}\cdot\vec{S} = \frac{b}{2}(J^2 - L^2 - S^2)$ where $\vec{J} = \vec{L} + \vec{S}$

$\underbrace{L^2}_{l=0,1,2}$, $\underbrace{S^2}_{0,1}$

use l_{tot}, s_{tot} and j_{tot} to label states

antisymmetric states : $l=0, s=0$; $l=1, s=1$; $l=2, s=0$

(a) $b>0 \rightarrow$ minimize j and maximize l and s

$^3P_0$ $\hookrightarrow$ $l=1, s=1, j=0$ - not degenerate

(b) $b<0$, $l=0, s=0, j=0$ and $l=2, s=0, j=2$

become degenerate excited states, but the gd. state is $j=2, l=1, s=1$

$^3P_2$