PHYSICS 632 Winter 2005 Jan. 27 midterm - CORRECTED

Name:____

1) A particle in a sphere, $r < a$, is in the state:

$\psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{2}}[\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})]$,

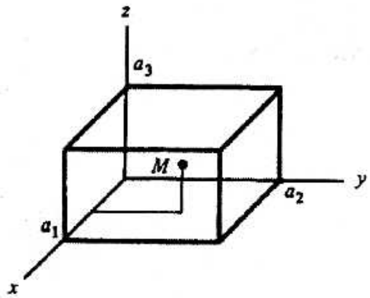
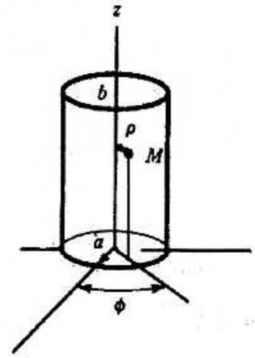
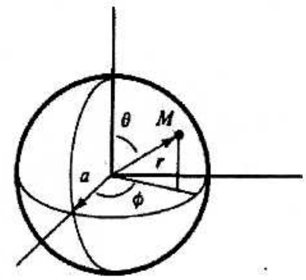
where ϕ_n are the energy eigenstates, with energies E_n, ordered from lowest eigenvalue to highest. The first state is the $l = m = 0$ ground state. (a) What is the lowest energy that can be measured? Assume mass M and give the answer in terms of M and a. (b) What is $\psi(x, t)$? (c) What is $< \hat{R} > (t)$, where $< r|\hat{R}|f >= rf(r)$. Write your answer in terms of well-defined radial integrals.

2) $\hat{H} = \frac{\hat{L}^2}{2I} + \omega_0 \hat{L}_z$. What are the eigenstates and eigenvalues of $\hat{H}$? At $t = 0$ the particle is in the lowest energy eigenstate with $l = 1$. What is $< \hat{L}_x > (t)$.

3) (a) Compute $[\hat{L}_i, \hat{R}^2]$. (b) If $\hat{\mathbf{L}} = \hat{\mathbf{L}}_\mathbf{1} + \hat{\mathbf{L}}_\mathbf{2}$, what is $[\hat{L}^2, \hat{\mathbf{L}}_\mathbf{1} \cdot \hat{\mathbf{L}}_\mathbf{2}]$? Show your work.

4) For angular momentum one-half there are two eigenstates, $|+>$ and $|->$, where $J^2|\pm>= 3/4|\pm>$ and $J_z|\pm>= \pm 1/2|\pm>$. $H = \omega_0 J_z$. At $t = 0$, $|\psi(t = 0)>= \frac{1}{\sqrt{2}}\big(|+> + |->\big)$. What is $|\psi(t)>$? What are the possible measurements of J_x? What are the probabilities of each as a function of time?

TABLE 10.2 Solutions to the three fundamental box problems in quantum mechanics

The Rectangular Box	The Cylindrical Box	The Spherical Box
Edge Lengths a_1, a_2, a_3	Radius a, Height b	Radius a
		
Hamiltonian		
$\hat{H} = (\hat{p}_x{}^2 + \hat{p}_y{}^2 + \hat{p}_z{}^2)/2M$	$\hat{H} = (\hat{p}_\rho{}^2 + \hat{p}_z{}^2 + \hat{L}_z{}^2/\rho^2)2M$	$\hat{H} = (\hat{p}_r{}^2 + \hat{L}^2/r^2)/2M$
$\hat{p}_x{}^2 = \left(-i\hbar\frac{\partial}{\partial x}\right)^2$	$\hat{p}_\rho{}^2 = -\hbar^2\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)$	$\hat{p}_r{}^2 = -\hbar^2\left(\frac{1}{r}\frac{\partial}{\partial r}r\right)^2$
Eigenfunction		
$\varphi_{qst} = A_{qst}\sin k_q x \sin k_s y \sin k_t z$	$\varphi_{qmn} = A_{qmn} J_m(K_{mn}\rho)\sin k_q z e^{im\phi}$	$\varphi_{nlm} = A_{nlm} j_l(k_{ln}r) Y_l^m(\theta,\phi)$
$(A_{qst})^2 = 8 \neq a_1 a_2 a_3$	$(A_{qmn})^2 = 2/\pi b[a J_m{}'(K_{mn}a)]^2$	$(A_{nlm})^2 = 2/a^3[j_l{}'(k_{ln}a)]^2$
$\sin k_q a_1 = \sin k_s a_2 = \sin k_t a_3 = 0$	$\sin k_q b = J_m(K_{mn}a) = 0$	$j_l(k_{lna}) = 0$
Wave Equation	*Bessel's Equation*	*Spherical Bessel Equation*
$\left(\frac{d^2}{dx^2} + k^2\right)\sin kx = 0$	$\left[\frac{1}{x^2}\left(x\frac{d}{dx}\right)^2 + 1 - \frac{m^2}{x^2}\right]J_m(x) = 0$	$\left[\left(\frac{1}{x}\frac{d}{dx}x\right)^2 + 1 - \frac{l(l+1)}{x^2}\right]j_l(x) = 0$
Eigenenergy		
$E_{qst} = \hbar^2(k_q{}^2 + k_s{}^2 + k_t{}^2)/2M$	$E_{qmn} = \hbar^2(K_{mn}{}^2 + k_q{}^2)/2M$	$E_{nl} = \hbar^2 k_{ln}{}^2/2M,\ ak_{ln} \equiv x_{ln}$

$$\frac{1}{R}(r^2R'' + rR') + r^2(k^2 - k_z{}^2) = m^2 \tag{10.77c}$$

where a prime denotes differentiation with respect to r. With conditions (10.74) we find

$$Z(z) = A\sin k_z z, \qquad k_z L = n_z\pi, \qquad n_z = 1, 2, \ldots \tag{10.78a}$$

$j_\ell(X_{\ell n}) = 0$;

ℓ, n	0,1	1,1	2,1	0,2
$X_{\ell n}$	π	4.49	5.76	2π

$j_0(x) = \frac{\sin(x)}{x}$, $j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$

$Y_0^0 = \sqrt{\frac{1}{4\pi}}$, $Y_1^1 = -\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2}\sin\theta\, e^{i\phi}$, $Y_1^0 = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2}\cos\theta$, $Y_1^{-1} = \frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2}\sin\theta\, e^{-i\phi}$