

Two-body boson and fermion states:

To isolate statistics use one dimension, where there is no spin. Particle in a box to leading approximation, so this is a simplified version of particles with spin confined to a sphere in three dimensions.

generic problems

- 1) Find the first 4 energies and eigenstates of 3 identical bosons in a 1d well.
- 2) Same thing for 3 identical fermions.

Even though there is no spin, we can give the particles other properties. The simplest thing to do is invent something, call it Flavor, and make it work exactly like spin. Isospin, since it is isomorphic to spin.

Add $H_I = \hbar\omega_T \vec{T}_1 \cdot \vec{T}_2$, where $\tau_{ij} = \sigma_{ij}$ are Pauli spin matrices.

↙ This changes the level structure.

tough problem

- 3) How does the level structure change for the 1st three states of the 2-fermion system, as ω_T varies:

$$E_n = n^2 \hbar\omega_0 + \hbar\omega_T \vec{T}_1 \cdot \vec{T}_2$$

spatial states are particle-in-a-box states labelled by n

↑
box spectrum

↑
 • $\omega_T \ll \omega_0$
 • $\omega_T \gg \omega_0$
 $\omega_T \sim \omega_0$

Electrons confined to a sphere

$$\hat{H} = \left(\hat{p}_r^2 + \frac{\hat{L}^2}{r^2} \right) / 2M$$

$$p_r^2 = -\hbar^2 \left(\frac{1}{r} \frac{\partial}{\partial r} \right)^2$$

$$\phi_{n\ell m} = A_{n\ell m} j_\ell(k_{n\ell} r) Y_\ell^m(\theta, \phi)$$

confinement
bdr. cond.

$$j_\ell(k_{n\ell} a) = 0$$

$$E_{n\ell} = \frac{\hbar^2 k_{n\ell}^2}{2M}, \quad X_{n\ell} = k_{n\ell} a$$

$$\int_0^\infty r^2 dr j_\ell(kr) j_\ell(k'r) = \frac{\pi}{2k^2} \delta(k-k') \quad \left\{ \begin{array}{l} \text{note: this holds} \\ \text{only for same } \ell \end{array} \right.$$

shell →	ℓ, n	0, 1	1, 1	2, 1	0, 2	3, 1	1, 2
leading spectrum →	$X_{\ell n}$	π	4.49	5.76	2π	6.99	7.73
# electrons →		2	6	10	2	14	6
level name →		1s	1p	1d	2s	1f	2p

⇒
TYPICAL
PROBLEM
type

Find the shell configuration (e.g., $(1s)^2(1p)^6(1d)^4$) and the L_S for the top shell electrons (e.g., $5D_0$).

The first step is easy, just fill the shells in order.

The second step is ruled by additional interactions involving angular momentum and spin; e.g.,

$$\hat{H} = b \hat{S}_1 \cdot \hat{S}_2 \quad | \quad \hat{H} = b' \hat{L}_1 \cdot \hat{L}_2 \quad | \quad H = b'' (S_1 \cdot L_2 + S_2 \cdot L_1)$$

$$H = b''' (S_1 \cdot L_1 + S_2 \cdot L_2) \quad | \quad H = c J_1 \cdot J_2$$

Sample questions (electrons confined to a sphere)

- 1) $H = c \mathbf{J}_1 \cdot \mathbf{J}_2$. What is the shell configuration and spectroscopic notation for the 4 electron ground state ?
- 2) $H = b \mathbf{S}_1 \cdot \mathbf{S}_2$ - same question
- 3) $H = b' \mathbf{L}_1 \cdot \mathbf{L}_2$ - same question
- 4) $H = \sum_i \zeta_i \mathbf{L}_i \cdot \mathbf{S}_i$ - every electron experiences a spin-orbit interaction

If there are three electrons, what fraction of the electric charge is inside the $r < \frac{a}{2}$ sphere given that all of the charge lies in the $r < a$ sphere.