PHYSICS 632 Winter 2007 Exam #2 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer.

1)Two particles are in a one-dimensional well of width a. (a) If the particles are identical bosons with no "spin", what are the energies of the ground state and first excited state and what is the degeneracy of each of these states? What is the first excited state wave-function written using the one-body particle-in-a-well eigenstates? If there is more than one possible first excited state, pick one and tell me which one you are picking. (b) Same problem for two identical fermions with no spin.

2) Two spin-1 bosons are confined to a sphere of radius a. (a) What is the ground state wave function $\psi(\mathbf{r_1}, \mathbf{r_2}, s_1, s_2)$? Show the radial, angular and spin parts of the wave-function, and since there are many possiblities you must pick one. What is the degeneracy of the ground state? (b) If the interaction $\delta H = b\mathbf{S_1} \cdot \mathbf{S_2}$, with b positive, is added, the ground state level is split into many levels. What are the energies of these levels and what quantum numbers label them?

3) A spin-1/2 particle has $H = H_0 + H'$ with $H_0 = \omega_0 S_z$ and $H' = \omega_1 S_y$. Treating H' as a perturbation compute the first order shifts in the energies of the spin-up and of the spin-down states using first-order perturbation theory. Compute the exact eigenvalues and compare with your perturbative result.

4) H_0 is the Coulomb hamiltonian for hydrogen. $H' = -eE_0 z$. Compute the first-order energy shift for the $n = 1$ state and for a complete set of $n = 2$ states. (Note: You may not be able to do the whole problem.)

5) H_0 is the 2-dimensional well, with $0 < x < a$ and $0 < y < a$. $H' = \alpha\delta(x - a/2)\delta(y - a/2)$. (a) Compute $E^{(1)}_{mn}$. (b) Demonstrate whether $E^{(2)}$ is finite or not.

6) A particle with spin 1 ($s_1 = 1$) interacts with a particle with spin 1/2 ($s_2 = 1/2$). The hamiltonian is:

$$H = \lambda\, \mathbf{S_1} \cdot \mathbf{S_2}$$

(a) Give a complete set of eigenstates (show all quantum numbers and say what each is in words; e.g., s is the total spin) and the energy eigenvalue for each state.

(b) At time $t = 0$ the z-component of the first particle's spin is measured to be 0 and the z-component of the second particle's spin is measured to be 1/2. At a later time t_1, the z-component of the first particle's spin is measured again. What are the possible results and their respective probabilities?

7) A particle is confined to an infinite one-dimensional square well so that the wave function is non-zero only for $0 < x < a$. A perturbing potential, $V(x) = -\alpha\, \delta(x - a/2)$, is added. (a) Compute the first-order shift in all energies. (b) Compute the second-order shift in the ground state energy, leaving your answer as a sum but clearly stating or showing which terms are zero (if any). Is this second-order shift finite or infinite?

8) Two non-identical spin-1/2 particles ($s_1 = 1/2$, $s_2 = 1/2$) are fixed in space, so that we only need their spin state. To leading order:

$$H_0 = \omega\, \mathbf{S_1} \cdot \mathbf{S_2},$$

where $\omega > 0$. (a) What are the eigenstates and eigenvalues? (b) A perturbing potential, $V = \lambda S_{1z}$, is added. Note that only the first particle is involved in V. Compute the first-order perturbative shifts of the energies for each eigenstate. (c) Compute the second-order shift in the energy of the ground state.