

PHYSICS 631 Autumn 2006 Midterm #3 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer.

This test covers the infinite square well, the harmonic oscillator, the free particle, the δ -function potential, the finite square well and Dirac notation. Review sample problems for and problems from the first two tests. You should know how to do problems 2.40, 2.47, 3.27, 3.37, and 3.38 in Griffiths. Figuring out how to do these problems does not require actually doing all of them, which would take far more time than convincing yourself you know how to do them.

Additional problems (most from old exams):

1) A particle of mass m is bound in the potential $V(x) = \infty$ for $x < 0$, $V(x) = -V_0$ for $0 \leq x \leq a$ and $V(x) = 0$ for $x > a$. (a) Find the wave function in each region for the ground state. Use the boundary conditions to find a transcendental equation for the energy. You may leave one overall constant in the wave function undetermined (i.e., you do not need to normalize the wave function). Express all constants in terms of m, V_0, a, E , where E is the energy. (b) There are two bound states in this well. This means that V_0 must lie between two constants that are determined by m, a, \hbar . What are these bounds on V_0 if there are two and only two bound states?

2) A particle of mass m approaches the step potential $V(x) = 0$ for $x > 0$ and $V(x) = V_0$ for $x < 0$ from the right. It has an energy $2V_0$. What is the probability that it will be reflected? How does this probability change as the energy is lowered to zero?

3) There are two bound states in the potential $V(x) = 0$ for $x < -a$, $V(x) = -V_0$ for $-a < x < 0$, and $V(x) = \infty$ for $x > 0$. The second bound state is barely bound, with binding energy very near zero. Find the wave function in each region and set up the boundary conditions. Specify any constants you use in terms of m, a and V_0 . Use the boundary conditions to determine V_0 in terms of m and a , given the fact that there are only two bound states. Sketch the two bound states.

4) There are three bound states in the potential $V(x) = 0$ for $x < -a$, $V(x) = -V_0$ for $-a < x < a$, and $V(x) = \infty$ for $x > a$. The third bound state is barely bound, with binding energy very near zero. Find the wave function in each region and set up the boundary conditions. Specify any

constants you use in terms of m , a and V . Use the boundary conditions to approximate V_0 in terms of m and a , given the fact that there are only three bound states and the third is barely bound. Sketch the three bound states.

5) The finite square well potential $V(x) = 0$ for $|x| > a$ and $V(x) = -V_0$ for $|x| < a$ has two antisymmetric bound states and the second is just barely bound with $|E| \ll |V_0|$. How many bound states are there in total? Sketch each and approximate their energies.

6) Particles are shot “from the right” at the potential $V(x) = \infty$ for $x < 0$, $V(x) = -V_0$ for $0 \leq x \leq a$, and $V(x) = 0$ for $x > a$, so that the wavefunction for $x > a$ is $e^{-ikx} + Be^{ikx}$. Assume $E = 2V_0$, compute B and compute the wavefunction inside the well by setting up and solving the boundary conditions at $x = a$. Express all constants you use in terms of E , V_0 and m .