PHYSICS 631 Autumn 2006 Midterm #2 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer.

This test covers the infinite square well, the harmonic oscillator, the free particle and the δ-function potential. Review sample problems for and problems from the first test. You should know how to do problems 2.36, 2.38, 2.41, 2.42, 2.44, 2.46 and 2.48 in Griffiths. Figuring out how to do these problems does not require actually doing all of them, which would take far more time than convincing yourself you know how to do them.

Additional problems (most from old exams):

1) A particle of mass m is free, $\hat{H} = \hat{P}^2/(2m)$. At $t = 0$ the particle is in the initial state $< x|\psi(0) >= N\theta(x)\theta(a-x)$, a constant from $x = 0$ to $x = a$. (a) What is $< x|\psi(t) >$ for later times? Evaluate any integrals you encounter and fix N so that the state is normalized. (b) If momentum is measured at time t, what is the probability per unit of momentum for finding momentum p. (c) If energy is measured at time t, what are the possible results and what is the probability for 'each' result?

2) For a free particle, $\hat{H} = \frac{\hat{P}^2}{2m}$. At $t = 0$, $\psi(x) = A \exp[\frac{-x^2}{L^2}]$. Note that $\int_{-\infty}^{\infty} dx \exp[-a\, x^2] = \sqrt{\pi/a}$. (2a) What is $\psi(x,t)$? You may leave integrals unevaluated but be careful to specify limits. (2b) What are $< \hat{P} > (t)$ and $< \hat{H} > (t)$? You should be able to completely evaluate any integrals in this part.

3) For a free particle, $\hat{H} = \frac{\hat{P}^2}{2m}$. At $t = 0$, $\Psi(x,0) = A \sin(\frac{2\pi x}{L})$ for $-L < x < L$. (a) What is $\psi(x,t)$? (b) What are $< \hat{P} >$ and $< \hat{H} >$?

4) $V(x) = -\alpha\, \delta(x) + \alpha\, [\delta(x-a) + \delta(x+a)]$, where $\alpha > 0$. Find the ground state wave function and ground state energy. Use the fact that $V(-x) = V(x)$ to reduce the number of constants that must be found and the number of simultaneous equations that must be solved. Is there a second bound state?