PHYSICS 631 Autumn 2006 Midterm #1 Sample Problems from old tests

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Don't worry if you are unfamiliar with the Dirac notation. These are questions from the last two years of Physics 631.

1) A particle of mass m is confined to a "box" in one dimension which goes from $x = 0$ to $x = a$. The energy eigenstates satisfy $\Psi_n(x,t) = \psi_n(x)\, e^{-i\omega_n t}$. In this box $\psi_n(x) = \sqrt{2/a}\,\sin(k_n x)$, where $k_n = n\pi/a$ and the energy is $\hbar\omega_n = \hbar^2 k_n^2/(2m)$. (a) For a particle in the first excited state ($n = 2$), what is the probability of finding the particle in the left half of the box ($0 < x < a/2$)? What are the possible values of momentum that can be found? (b) For a particle in the state $\Psi(x,t) = \frac{1}{\sqrt{2}}\Big[\Psi_1(x,t) + \Psi_4(x,t)\Big]$, compute $< x >$. You do not need to explicitly evaluate integrals that *are not zero*, but if an integral vanishes, state this and drop it from the final answer.

2) A particle of mass m is confined to a "box" in one dimension which goes from $x = -a$ to $x = a$. (a) What are the energy eigenstates and their energies? Normalize the states and specify what values any integer index you use can have. (b) A particle is in the state $1/\sqrt{2}\big[\psi_{gd}(x)e^{-i\omega_{gd}t} + \psi_{1st}(x)e^{-i\omega_{1st}t}\big]$. Set up the expression for $< x > (t)$ and use symmetry to eliminate all integrals that go to zero. You do not need to evaluate the remaining integrals.

3) A particle of mass m is bound in an infinite square well so that it is free for $0 < x < a$ and unable to leave this region. At time $t = 0$, it is in a simple state localized around the middle of the box at $x = a/2$. At $t = 0$, $< x|\psi(0) >= N\theta(x - a/4)\theta(3a/4 - x)$. In words, it is equal to N from $x = a/4$ to $x = 3a/4$ and it is zero everywhere else. (a) What is $< x|\psi(t) >$ for later times? Evaluate any integrals you encounter and fix N so that the state is normalized. (b) If energy is measured at time t, what are the possible results and what is the probability for each result?

4) A particle in a box, $-a < x < a$, is in the state: $\psi(x, t = 0) = \frac{1}{\sqrt{2}}[\phi_1(x) - \phi_2(x)]$, where ϕ_n are the energy eigenstates, with energies E_n. (a) What is the lowest energy that can be measured? Assume mass m and give the answer in terms of m and a. What is $\psi(x,t)$? (b) Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$ and verify Ehrenfest's principle.

5) A particle of mass m is bound by a harmonic oscillator potential. See the handout for all details that you need. (a) In this part, $< x|\psi(0) >= N\exp(-x^2/a^2)$, which has a different width from the ground state. What is $< x|\psi(t) >$? Your answer can include integrals that are not evaluated, but everything in your expression should be completely defined. Choose N to normalize the state. (b) In this part $|\psi(0) >= 1/\sqrt{2}\Big(|\psi_0 > +|\psi_1 >\Big)$. Here, $|\psi_0 >$ is the ground state and $|\psi_1 >$ is the first excited state. What is $< p|\psi(t) >$? Evaluate any integrals you encounter.

6) (a) Compute $< \hat{X}\hat{P} >$ and $< \hat{P}\hat{X} >$ for a particle in harmonic oscillator state $|\phi_n >$. Define any constants you use in terms of m and K, where, $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K}{2}\hat{X}^2$. (b) Compute $< \hat{X}\hat{P} >$ for a state that is $|\psi(t=0) >= \frac{1}{\sqrt{2}}|\phi_0 > + \frac{1}{\sqrt{2}}|\phi_1 >$ at time $t = 0$. This is a function of time. The algebraic method is the easiest way to do this calculation.

7) At $t = 0$ a particle is placed in a state $|\psi(t=0) >= \frac{1}{\sqrt{2}}|\phi_1 > - \frac{1}{\sqrt{2}}|\phi_2 >$, where $|\phi_n >$ are eigenstates of the harmonic oscillator Hamiltonian, $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K}{2}\hat{X}^2$.
(a) Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$ for $|\psi(t) >$. Define any constants you use in terms of m and K. (b) Compute $\frac{d<\hat{X}>}{dt}$ and $\frac{d<\hat{P}>}{dt}$, and discuss whether these satisfy Ehrenfest's principle.

8) At $t = 0$ a particle in a harmonic oscillator potential is placed in a state $|\psi(t=0) >= \frac{1}{\sqrt{2}}|\phi_3 > - \frac{1}{\sqrt{2}}|\phi_1 >$, where $|\phi_n >$ are eigenstates of the harmonic oscillator Hamiltonian, $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K}{2}\hat{X}^2$. (a) What is $|\psi(t) >$? Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$. Define any constants you use.

9) At $t = 0$ a particle in a harmonic oscillator potential is placed in a state $|\psi(t=0) >= \frac{1}{\sqrt{2}}|\phi_2 > - \frac{1}{\sqrt{2}}|\phi_0 >$, where $|\phi_n >$ are eigenstates of the harmonic oscillator Hamiltonian, $\hat{H} = \frac{1}{2m}\hat{P}^2 + \frac{K}{2}\hat{X}^2$. (9a) What is $|\psi(t) >$? Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$. Define any constants you use. (9b) Compute $< \hat{X}^2 > (t)$ and $< \hat{P}^2 > (t)$.