PHYSICS 631 Autumn 2006 Midterm #3

Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) A particle of mass m is "in" a harmonic oscillator potential, $V(x) = \frac{1}{2}m\omega^2 \, x^2$. It starts at $t = 0$ in the state:

$$|\psi(t=0)> = \sqrt{\tfrac{1}{2}}\left[|\psi_2> - |\psi_3>\right].$$

(a) If the energy is measured at time t, what is the average value? (b) Compute $< X >$, $< X^2 >$, $< P >$ and $< P^2 >$ at time t. Note that you can set up the generic calculation in Dirac notation and then need very little space to compute the particular matrix elements required for each of these expectation values. (c) Check Ehrenfest's Principles and the uncertainty principle.

2) A particle with energy $E = \frac{k^2}{2m}$ approaches the step potential drawn below *from the right, from positive infinity.* The potential $V(x) = 0$ for $x > 0$ and $V(x) = -V_0$ for $x < 0$, so the particle "sees" the potential step *down.* Compute R and T, the probability that it will reflect and the probability that it will pass over the step and be transmitted to $x \to \infty$.

3) $\hat{H} = \epsilon|1><1| - \epsilon|2><2| + i\epsilon|1><2| - i\epsilon|2><1|$, where $|1>$ and $|2>$ are basis states in our 2-dimensional space of states. (a) Find the eigenvalues (i.e., energies) and eigenstates of $\hat{H}$ in terms of this basis.

(b) $\hat{Q}$ is another "observable" for this system, for which the eigenstates are $|1>$ and $|2>$, with eigenvalues q_1 and q_2 respectively. If Q is measured for the lowest energy eigenstate of $\hat{H}$ (i.e., the ground state), what are the possible results and what is the probability of each?

(c) Suppose Q is measured at time $t = 0$ and the result is q_1. What is the state, $|\psi(t)>$, for times after this?