PHYSICS 631 Autumn 2006 Midterm #2

Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) A particle of mass m is confined to: $0 < x < a$. It starts at $t = 0$ in the state:

$$\psi(x) = \sqrt{\tfrac{4}{a}}\,\sin\left(\tfrac{2\pi x}{a}\right)\,\theta\left(\tfrac{a}{2} - x\right);$$

(a) What is $\Psi(x,t)$? (b) What is $< X >$ at time t? (c) What is $< H >$? Note: Your answers in (a) and (b) can include sums that are not evaluated, but your answer to (c) should not include unevaluated sums. Use the table of integrals in the notes to evaluate all integrals.

2) A particle of mass m is "in" a harmonic oscillator potential, $V(x) = \frac{1}{2}\mathrm{m}\omega^2\, x^2$. It starts at $t = 0$ in the state:

$$\psi(x) = \sqrt{\tfrac{1}{2}}\,\Big[\psi_1 - i\psi_3\Big].$$

(a) If the energy is measured at time t, what is the average value? (b) Compute $< X^2 >$ and $< X >$ and then compute $\sigma_x = \sqrt{< X^2 > - < X >^2}$? (c) Computer $< P^2 >$ and $< P >$ and use these to compute σ_p. (d) Is the uncertainty principle satisfied?

3) Particle in a box with a δ-function potential. Assume $E = \frac{\hbar^2 k^2}{2m}$.

$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{box}(x) + \alpha\delta(x)$, where $V_{box}(x) = 0$ for $-a/2 < x < a/2$ and $V_{box}(x) = \infty$ for all other x. Assume $\alpha > 0$ and note that this is a repulsive δ-function and use the notes. (a) Solve $H\psi(x) = E\psi(x)$ in the regions left and right of $x = 0$, $\psi_L(x)$ and $\psi_R(x)$. Use the boundary conditions $\psi_L(-a/2) = \psi_R(a/2) = 0$ to remove all but one arbitrary constant in each of these solutions. (b) Consider the ground state now. Use the two δ-function boundary conditions at $x = 0$ to find an equation for k. Either solve this equation or use a plot that shows how you could find the solution numerically. In particular, look at the ground state solution, the lowest value of k that solves the equation. Consider what happens as $\alpha \to 0$ and as $\alpha \to \infty$. You should be able to solve the equation in these limits and check that the answer makes sense.