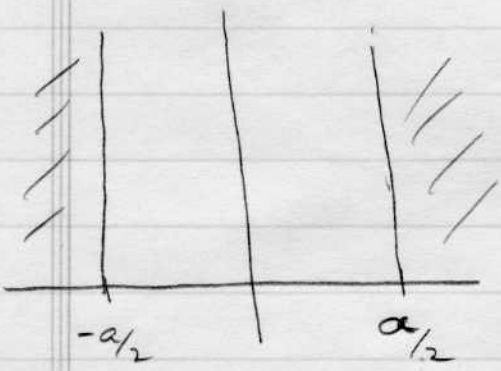
Physics 631 AU05

Sample midterm 2 problems

1) Infinite square well



$$V(x) = \begin{cases} \infty & x < -a/2 \\ 0 & -a/2 < x < a/2 \\ \infty & x > a/2 \end{cases}$$

(a) Compute the complete set of normalized eigenstates and their energies.

(b) At $t=0$, $\Psi(x, t=0) = \frac{1}{\sqrt{2}}\left(\psi_{gd}(x) + \psi_{1st}(x)\right)$,

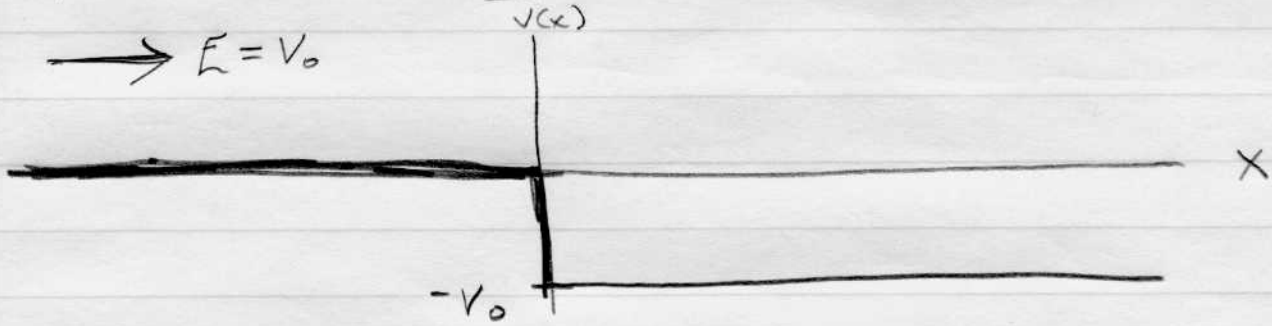
where $\psi_{gd}(x)$ is the ground state wave function and $\psi_{1st}(x)$ is the 1st excited state wave function.

What is $\Psi(x,t)$ for $t>0$?

(c) Using $\Psi(x,t)$ from (b), compute $\langle x \rangle(t)$ and $\langle p \rangle(t)$.

2) Scattering

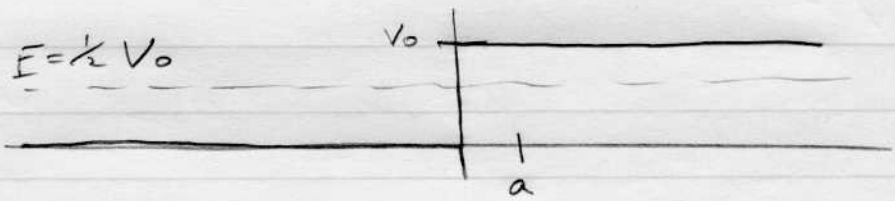
a)



$$V(x) = \begin{cases} 0 & x < 0 \\ -V_0 & x > 0 \end{cases}$$

A particle with energy $+V_0$ approaches the step shown above from $x = -\infty$. What is the probability that it will go past the step towards $x = +\infty$?

b)



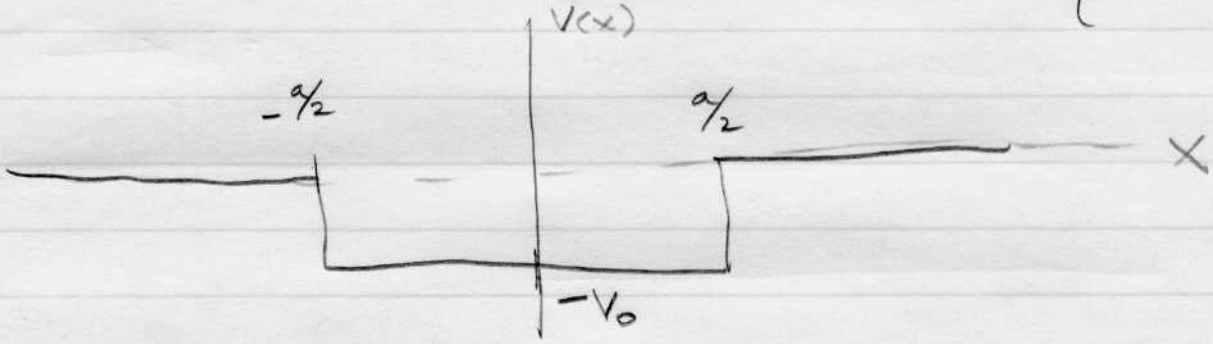
$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases}$$

A current of particles with energy $+\frac{1}{2}V_0$ approaches the barrier above from the left. Call:

$$\psi(x) = e^{ikx} + Be^{-ikx} \quad \text{for } x < 0,$$

what is the probability per unit length at $x = a$ of finding a particle?

3) Finite Square Well $V(x) = \begin{cases} 0 & |x| > a/2 \\ -V_0 & |x| < a/2 \end{cases}$



a) Given the mass m, and distance a for the symmetric square well above, how large must V_0 be for one bound state to exist? How deep must it be for two bound states to exist?

b) Assume there are three bound states; sketch them.

4) Harmonic Oscillator $V(x) = \frac{1}{2}Kx^2$

The ground state is $\Psi_0(x,t)$ and the first excited state is $\Psi_1(x,t)$. The state for a particle in this potential is:

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left[\Psi_0(x,t) - \Psi_1(x,t)\right]$$

a) Sketch the wave-function at $t=0$.

b) Compute $\langle x\rangle(t)$ and $\langle x^2\rangle(t)$.

c) Compute $\langle p\rangle(t)$ and $\langle p^2\rangle(t)$.

d) Does $\frac{d}{dt}\langle p\rangle = -K\langle x\rangle$?