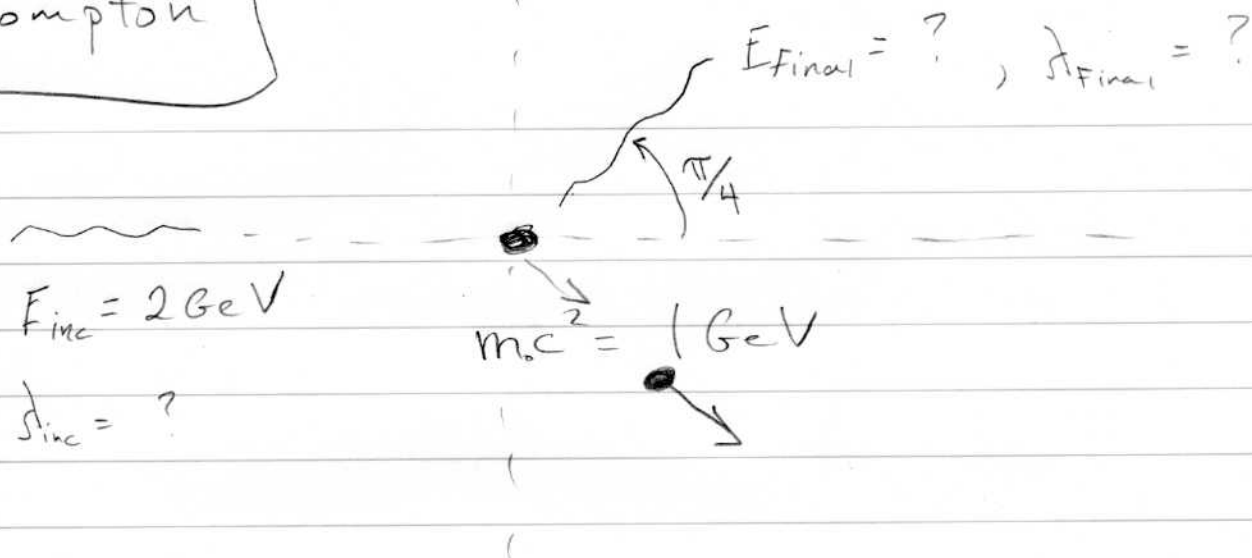
Compton



A 2 GeV photon collides with a $1\,GeV/c^2$ mass particle initially at rest. It scatters at an angle of $\pi/4$. What is the final energy of the photon? What are its initial and final wavelengths? Can we use a nonrelativistic approximation for the scattered particle?

The algebra simplifies for some angles. Another interesting case is π.

Bohr Atom

For hydrogen we have a charge e proton and a charge e^- electron. If we completely ionize an atom with nuclear electric charge Ze (i.e., we have Z protons) so that only one electron remains, we have:

$$E_n = \frac{p_n^2}{2m_e} - \frac{Z\alpha\hbar c}{r_n}$$

$$\alpha = \frac{1}{137} \qquad \hbar c = 197.3 \text{ MeV-Fm}$$
$$= \frac{1}{2\pi}\left(12345 \text{ eV-Å}\right)$$

$$m_e c^2 = 0.511 \text{ MeV}$$

Use Bohr's quantization condition to solve for:

$$E_n,\ r_n,\ v_n = \frac{p_n}{m_e}$$

How large does Z have to be for relativity to become important?

Complex Algebra

1) Write each of the following in polar coordinates, $re^{i\theta}$. Assume $0 \leq \theta < 2\pi$.

a) $2+i$ b) $3-2i$ c) $1-i$ d) i^2

2) Compute $|z|^2$ for a-d.

3) What is $z_1^* \cdot z_2$ for:

(a) $z_1 = 1+i$, $z_2 = 1-i$ (b) $z_1 = \sqrt{\pi} - i$, $z_2 = 1 + i\sqrt{\pi}$

(c) $z_1 = i$, $z_2 = 1$

4) Compute $1/z$ in polar coordinates for (a-b) in #1.

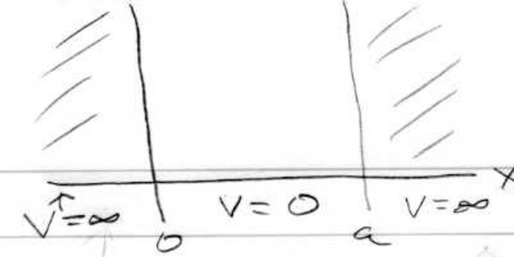
5) A time-dependent sq. well state is:

$$\psi(x,t) = \frac{1}{\sqrt{2}} \left\{ \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) e^{-i\omega_1 t} + \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) e^{i\omega_2 t} \right\}$$

where $\omega_1 = \frac{\hbar}{2m}\left(\frac{\pi}{a}\right)^2$, $\omega_2 = 4\omega_1$

Compute $\langle x(t) \rangle$.

Particle in a Box



$0 \le x \le a$, $i\hbar \frac{\partial}{\partial t} \Psi(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t)$

$$\Psi(0,t) = \Psi(a,t) = 0$$

energy eigenstate: $\Psi_n(x,t) = \psi_n(x)\, e^{-i\omega_n t}$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad , \; n = 1, 2, 3, \ldots$$

$$\omega_n = n^2 \omega_0 \;, \; \hbar\omega_0 = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2$$

1) For an electron ($m_e c^2 = 0.511$ MeV) in a 1 Å well, what is the ground state energy?

2) If $\Psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_1(x) e^{-i\omega_1 t} + \psi_2(x) e^{-i\omega_2 t}\right]$,

a) what is the probability of finding the particle between x and $x+dx$?

b) what is the probability of finding the particle in the left half of the box as a function of time?

c) If energy is measured, what are the possible results and what are their respective probabilities?

d) What is $\langle p(t) \rangle$? What is $\langle x(t) \rangle$?
Is $m\langle \dot{x} \rangle = \langle p \rangle$? Think about it.