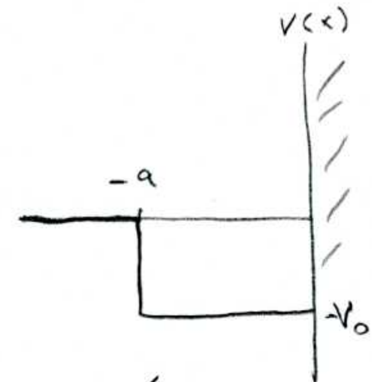
PHYSICS 631 **Autumn 2005** Midterm #2

Name:____

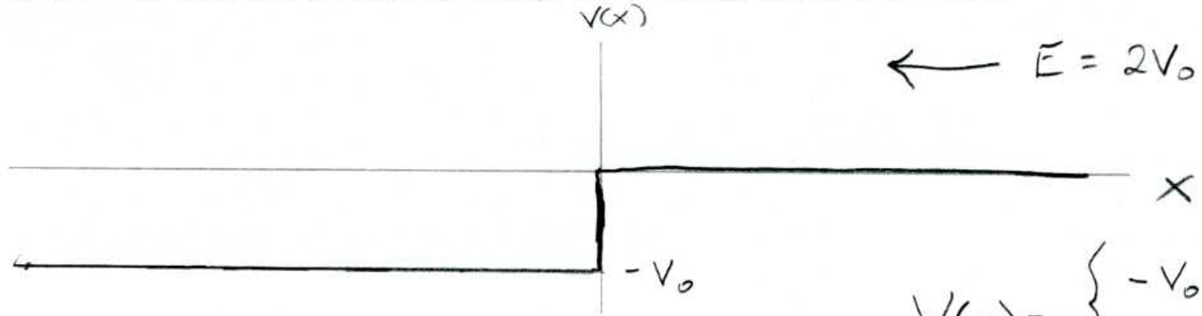
First solve each problem in the test booklet and then write the answer with all important steps on the test. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer, but only put your final solution on the test itself.

1) A particle of mass m is bound in the potential shown. (a) Find the wave function in each region for the ground state. Use the boundary conditions to find a transcendental equation for the energy. You may leave one overall constant in the wave function undetermined (i.e., you do not need to normalize the wave function). Express all constants in terms of m, V_0, a, E, where E is the energy. (b) There are two bound states in this well. This means that V_0 must lie between two constants that are determined by $m, a, \hbar$. What are these bounds on V_0 if there are two and only two bound states?



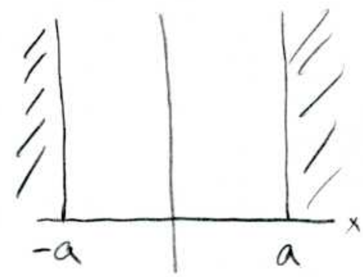
$$V(x) = \begin{cases} 0, & x < -a \\ -V_0, & -a < x < 0 \\ \infty, & x > 0 \end{cases}$$

2) A particle of mass m approaches the step potential sketched below from the right. It has an energy $2V_0$. What is the probability that it will be reflected? How does this probability change as the energy is lowered to zero?



$$V(x) = \begin{cases} -V_0, & x<0 \\ 0, & x>0 \end{cases}$$

3) A particle of mass m is confined to a "box" in one dimension which goes from $x = -a$ to $x = a$. (a) What are the energy eigenstates and their energies? Normalize the states and specify what values any integer index you use can have. (b) A particle is in the state $1/\sqrt{2}\left[\psi_{gd}(x)e^{-i\omega_{gd}t} + \psi_{1st}(x)e^{-i\omega_{1st}t}\right]$. Set up the expression for $< x > (t)$ and use symmetry to eliminate all integrals that go to zero. You do not need to evaluate the remaining integrals.



$$V(x) = \begin{cases} \infty, & x < -a \\ 0, & -a < x < a \\ \infty, & x > a \end{cases}$$