$$\hat{H} = \frac{\hat{p}^2}{2m} \qquad \text{Let } \hat{p}|k\rangle = \hbar k|k\rangle, \ \langle x|k\rangle = \frac{e^{ikx}}{\sqrt{2\pi}}$$

$$\langle x|\psi(0)\rangle = N\exp\left(-x^2/a^2 + ik_0x\right)$$

$$\langle\psi(0)|\psi(0)\rangle = N^2\int_{-\infty}^{\infty}dx\ \exp\left(-\frac{2x^2}{a^2}\right) = N^2\sqrt{\frac{\pi a^2}{2}}$$

$$\Rightarrow N = \left(\frac{2}{\pi a^2}\right)^{1/4}$$

$$\langle k|\psi(0)\rangle = \int_{-\infty}^{\infty}dx\ \langle k|x\rangle\langle x|\psi(0)\rangle$$

$$= N\int dx\ \frac{e^{-ikx}}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{a^2} + ik_0x\right)$$

$$-\frac{x^2}{a^2} - i(k-k_0)x = -\frac{1}{a^2}\left(x + \frac{i(k-k_0)a^2}{2}\right)^2 - \frac{a^2(k-k_0)^2}{4}$$

$$\langle k|\psi(0)\rangle = \frac{N}{\sqrt{2\pi}}\int dx\ \exp\left\{-\frac{1}{a^2}\left(x + \frac{i(k-k_0)a^2}{2}\right)^2 - \frac{a^2(k+k_0)^2}{4}\right\}$$

$$= \frac{N}{\sqrt{2\pi}}\exp\left(-\frac{a^2}{4}(k-k_0)^2\right)\sqrt{\pi a^2}$$

$$= \left(\frac{2}{\pi a^2}\right)^{1/4}\sqrt{\frac{a^2}{2}}\ \exp\left(-\frac{a^2}{4}(k-k_0)^2\right)$$

$$= \left(\frac{a^2}{2\pi}\right)^{1/4}\exp\left(-\frac{a^2}{4}(k-k_0)\right)^2$$

$$\langle x|\psi(t)\rangle = \ ?$$

$$\langle x|\psi(t)\rangle = \langle x| e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

$$= \int dk \ \langle x|k\rangle\langle k| e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

$$= \int dk \ \frac{e^{ikx}}{\sqrt{2\pi}} \ \exp\left[-i\frac{k^2 t}{2m\hbar}\right] \left(\frac{a^2}{2\pi}\right)^{1/4} \exp\left[-\frac{a^2}{4}(k-k_0)^2\right]$$

$$= \left(\frac{a^2}{(2\pi)^3}\right)^{1/4} \int dk \ \exp\left\{-\frac{a^2}{4}(k-k_0)^2 + ikx - i\frac{k^2 t}{2m\hbar}\right\}$$

$$\{\} = -\frac{a^2}{4}k^2 + \frac{a^2}{2}k_0 k - \frac{a^2}{4}k_0^2 + ikx - i\frac{k^2 t}{2m\hbar}$$

$$= \underbrace{\left(-\frac{a^2}{4} - \frac{it}{2m\hbar}\right)} k^2 + \underbrace{\left(\frac{a^2}{2}k_0 + ix\right)} k - \frac{a^2}{4}k_0^2$$

$$= -Ak^2 + Bk - \frac{a^2}{4}k_0^2$$

$$= -A\left(k - \frac{B}{2A}\right)^2 + \frac{B^2}{4A} - \frac{a^2}{4}k_0^2$$

shift in this variable

$$\langle x|\psi(t)\rangle = \left(\frac{a^2}{(2\pi)^3}\right)^{1/4} \exp\left\{\frac{\left(\frac{a^2}{2}k_0 + ix\right)^2}{4\left(\frac{a^2}{4} + \frac{it}{2m\hbar}\right)} - \frac{a^2}{4}k_0^2\right\}$$

$$\int dk \ \exp\left(-Ak^2\right)$$

$$\langle x|\psi(t)\rangle = \left(\frac{a^2}{(2\pi)^3}\right)^{1/4} \sqrt{\frac{\pi}{\frac{a^2}{4}+\frac{it}{2m\hbar}}}$$

$$\times \exp\left\{\frac{-\frac{x^2}{a^2}+ik_0x+\frac{a^2}{4}k_0^2}{1+\frac{2it}{m\hbar}} - \frac{a^2}{4}k_0^2\right\}$$