Harmonic Oscillator + Hermite Polynomials

$$\omega_0 = \sqrt{\frac{K}{m}} \quad ; \quad \frac{1}{2} K x^2 = \frac{1}{2} m \omega_0^2 x^2$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m \omega_0^2 x^2 \right] \psi_n(x) = E_n \psi_n(x)$$

$$\boxed{\begin{gathered} \text{Let } \beta = \sqrt{\frac{m\omega_0}{\hbar}} \quad ; \quad E_n = \left(n + \tfrac{1}{2}\right) \hbar \omega_0 \\ \psi_n(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\beta x)\, e^{-\beta^2 x^2/2} \end{gathered}}$$

$H_n(z)$ - Hermite polynomials

$$H_n(z) = \left(2z - \frac{d}{dz}\right)^n 1$$

$$H_0(z) = 1$$

$$H_1(z) = 2z$$

$$H_2(z) = 4z^2 - 2$$

$$H_3(z) = 8z^3 - 12z$$