

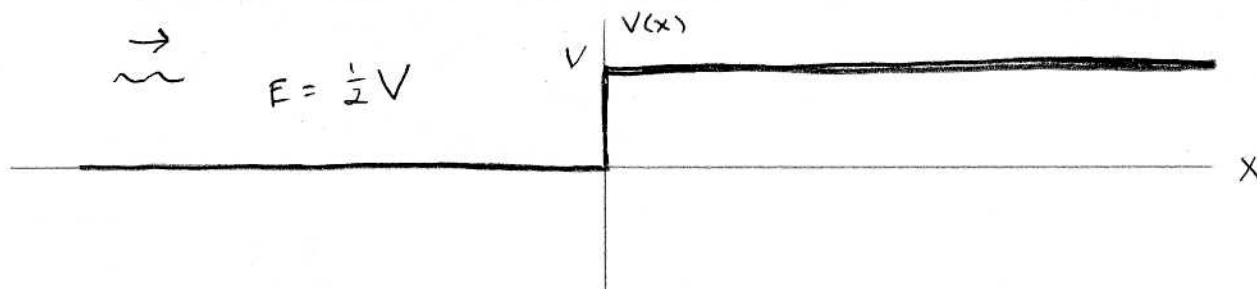
Name: \_\_\_\_\_

1) At  $t = 0$  a particle is placed in a state  $|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle - \frac{1}{\sqrt{2}}|\phi_2\rangle$ , where  $|\phi_n\rangle$  are eigenstates of the harmonic oscillator Hamiltonian,  $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K}{2}\hat{X}^2$ .

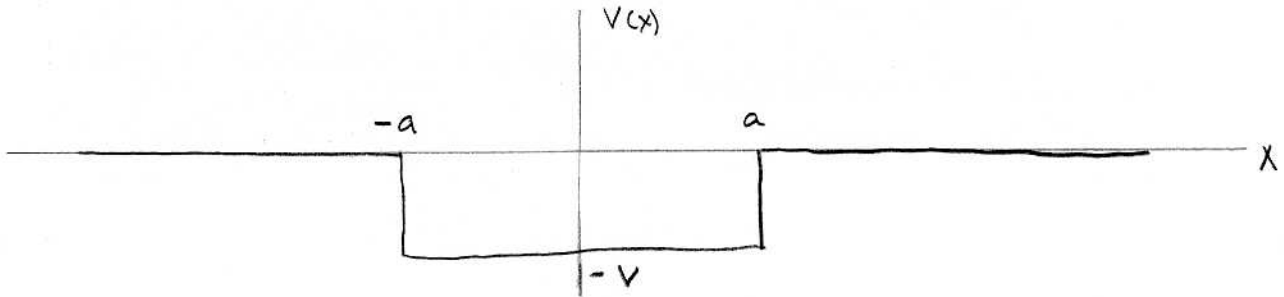
(a) Compute  $\langle \hat{X} \rangle (t)$  and  $\langle \hat{P} \rangle (t)$  for  $|\psi(t)\rangle$ . Define any constants you use in terms of  $m$  and  $K$ .

(b) Compute  $\frac{d\langle \hat{X} \rangle}{dt}$  and  $\frac{d\langle \hat{P} \rangle}{dt}$ , and discuss whether these satisfy Ehrenfest's principle.

2) Particles with  $E = V/2$  approach the step potential below from the left, so that  $|\psi(x)|^2 = e^{ikx} + Be^{-ikx}$  for  $x < 0$ . At what value of  $x$  does the probability density for finding particles drop to 1/2 the incoming density?



3) The finite square well potential shown below has two antisymmetric bound states and the second is just barely bound with  $|E| \ll |V|$ . How many bound states are there in total? Sketch each and approximate their energies.



4) Particles are shot at the well shown below from the right, so that the wavefunction for  $x > a$  is  $e^{-ikx} + Be^{ikx}$ . An infinite barrier prevents particles from entering the  $x < 0$  region, but  $V(x) = -V$  for  $0 < x < a$ . Assume  $E = 2V$ , compute  $B$  and compute the wavefunction inside the well by setting up and solving the boundary conditions at  $x = a$ . Express all constants you use in terms of  $E$ ,  $V$  and  $m$ .

