PHYSICS 631 Autumn 2004 Dec. 9 FINAL

Name:____

For a free particle, $\hat{H} = \frac{\hat{P}^2}{2m}$. At $t = 0$, $\psi(x) = A\ x\ \exp[\frac{-x^2}{L^2}]$. Note that $\int_{-\infty}^{\infty} dx\ x^2 \exp[-a\ x^2] = \sqrt{\pi}/(2a^{3/2})$. All integrals that you encounter can be completed using the table provided, but this is a difficult calculation and integral evaluation should be saved until you have finished all other problems. *You will receive all of the credit for this problem by setting up the correct integrals in part(a)*, but *you should be able to evaluate the integrals in part (b)*. You can shift arguments by imaginary amounts (e.g., x goes to x+iy) to obtain simple exponents as long as you do not move the contour of integration through a pole or cut, and these are not encountered here.

(1a) What is $\psi(x,t)$?

(1b) What are $< \hat{P} > (t)$ and $< \hat{H} > (t)$? You should be able to completely evaluate any integrals in this part. Use Ehrenfest's principle.

At $t = 0$ a particle in a harmonic oscillator potential is placed in a state $|\psi(t = 0) >= \frac{1}{\sqrt{2}}|\phi_2 > -\frac{1}{\sqrt{2}}|\phi_0 >$, where $|\phi_n >$ are eigenstates of the harmonic oscillator Hamiltonian, $\hat{H} = \frac{1}{2m}\hat{P}^2 + \frac{K}{2}\hat{X}^2$.

(2a) What is $|\psi(t) >$? Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$. Define any constants you use.

(2b) Compute $< \hat{X}^2 > (t)$ and $< \hat{P}^2 > (t)$.

3) There are three bound states in the potential well shown below. $V(x) = 0$ for $x < -a$, $V(x) = -V$ for $-a < x < a$, and $V(x) = \infty$ for $x > a$. The third bound state is barely bound, with binding energy very near zero. Find the wave function in each region and set up the boundary conditions. Specify any constants you use in terms of m, a and V. Use the boundary conditions to approximate V in terms of m and a, given the fact that there are only three bound states and the third is barely bound. Sketch the three bound states.