PHYSICS 631 **Autumn 2004** **Dec. 2 FINAL**

Name:____

1) A particle in a box, $-a < x < a$, is in the state:

$$\psi(x, t = 0) = \frac{1}{\sqrt{2}}[\phi_1(x) - \phi_2(x)],$$

where ϕ_n are the energy eigenstates, with energies E_n. (a) What is the lowest energy that can be measured? Assume mass m and give the answer in terms of m and a. What is $\psi(x, t)$?

(1b) Compute $<\hat{X}>(t)$ and $<\hat{P}>(t)$ and verify Ehrenfest's principle.

2) For a free particle, $\hat{H} = \frac{\hat{P}^2}{2m}$. At $t = 0$, $\psi(x) = A \exp[\frac{-x^2}{L^2}]$. Note that $\int_{-\infty}^{\infty} dx \exp[-a\ x^2] = \sqrt{\pi/a}$.

(a) What is $\psi(x,t)$? You may leave integrals unevaluated but be careful to specify limits.

(2b) What are $<\hat{P}>(t)$ and $<\hat{H}>(t)$? You should be able to completely evaluate any integrals in this part.

3) At $t = 0$ a particle in a harmonic oscillator potential is placed in a state $|\psi(t = 0) >= \frac{1}{\sqrt{2}}|\phi_3 > -\frac{1}{\sqrt{2}}|\phi_1 >$, where $|\phi_n >$ are eigenstates of the harmonic oscillator Hamiltonian, $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K}{2}\hat{X}^2$.

(a) What is $|\psi(t) >$? Compute $< \hat{X} > (t)$ and $< \hat{P} > (t)$. Define any constants you use.

(3b) Compute $< \hat{X}^2 > (t)$ and $< \hat{P}^2 > (t)$.

4) There are two bound states in the potential well shown below. $V(x) = 0$ for $x < -a$, $V(x) = -V$ for $-a < x < 0$, and $V(x) = \infty$ for $x > 0$. The second bound state is barely bound, with binding energy very near zero. Find the wave function in each region and set up the boundary conditions. Specify any constants you use in terms of m, a and V. Use the boundary conditions to determine V in terms of m and a, given the fact that there are only two bound states. Sketch the two bound states.