## van der Waals gas

$$Z_N = \frac{1}{N!}\left(\frac{V-bN}{\lambda_{th}^3}\right)^N e^{\beta a N^2/V}$$

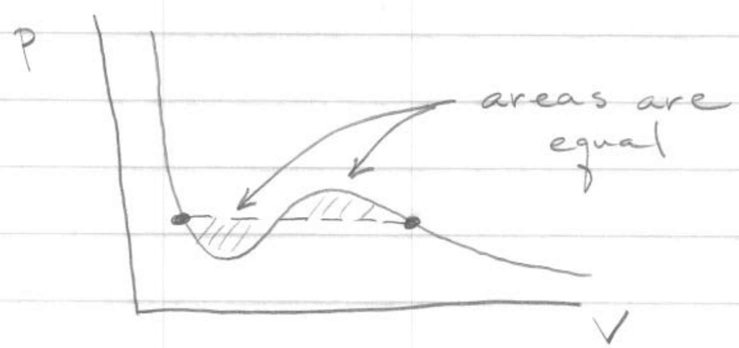
$$\left(p + \frac{aN^2}{V^2}\right)\left(V - bN\right) = NkT$$

$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$$

critical point $P_c, V_c, T_c$ found by fixing $T = T_c$

$$\left(\frac{\partial P}{\partial V}\right)_{T_c} = 0 \quad , \left(\frac{\partial^2 P}{\partial V^2}\right)_{T_c} = 0$$

Maxwell construction

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$$\Delta U = Q + W \quad , \quad W = -P\partial V$$

$$F = U - TS = -k_BT \ln Z$$

$$G = U - pV - TS = k_BT\left[-\ln Z + V\left(\frac{\partial \ln Z}{\partial V}\right)_T\right]$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_T$$

# Degeneracy Pressure

- pressure at $T = 0$ for fermions

$$\frac{N}{V} = n = \frac{2}{(2\pi)^3} \cdot 4\pi \int_0^{p_F/\hbar} q^2 \, dq \qquad \left\{ \boxed{n = \frac{1}{3\pi^2}\left(\frac{p_F}{\hbar}\right)^3} \right.$$

## nonrelativistic

$$E_F = \frac{p_F^2}{2m}$$

$$u = \frac{U}{V} = \frac{3}{5} n E_F$$

$$= \frac{3\hbar^2}{10m} (3\pi^2)^{2/3} n^{5/3}$$

$$P = \frac{2}{3} u = \frac{\hbar^2}{5m} (3\pi^2)^{2/3} n^{5/3}$$

## relativistic (extreme)

$$E_F = \sqrt{p_F^2 c^2 + m^2 c^4} \longrightarrow p_F c$$

$$u = \frac{3}{4} n E_F = \frac{3c\hbar}{4} (3\pi^2)^{1/3} n^{4/3}$$

$$P = \frac{1}{3} u = \frac{c\hbar}{4} (3\pi^2)^{1/3} n^{4/3}$$

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$$n_{e^-} = n_{proton} = \frac{Z}{A}\left(n_{proton} + n_{neuton}\right)$$

charge neutrality

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gravitational pressure $\Big\}$

$$P_{grav} = -\frac{G}{5}\left(\frac{4\pi}{3}\right)^{1/3} M^{2/3} \rho^{4/3}$$

Note: $n = \frac{\rho}{m}$