Physis 622 WI Ø8 2nd Exam Notes

partition function : $Z = \sum_i e^{-\beta E_i}$

$$U = -\frac{\partial}{\partial \beta} \ln Z \quad , \quad Z_N \simeq \frac{(Z_1)^N}{N!} \quad \text{- high } T$$

grand partition function: $\mathcal{Z} = \sum_\alpha e^{-\beta(E_\alpha - \mu N_\alpha)}$

no interaction : $\mathcal{Z} = \prod_k \mathcal{Z}_k = \prod_k \sum_{n_k} e^{-\beta(\epsilon_k - \mu) n_k}$

density of states $\sum_i \rightarrow \frac{(2S+1)V}{(2\pi)^3} \int d^3 q \rightarrow \int dE \, g(E)$

$$N = k_B T \left(\frac{\partial \ln \mathcal{Z}}{\partial \mu} \right)_{V,T} = \int_0^\infty dE \, g(E) \, f(E)$$

$$U = -\left(\frac{\partial \ln \mathcal{Z}}{\partial \beta} \right)_{\mu, V} + \mu N = \int_0^\infty dE \, g(E) \, f(E) \, E$$

$$f(E) = \frac{1}{e^{\beta(E-\mu)} \pm 1}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right), \quad \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \operatorname{arcsinh}\left(\frac{x}{a}\right), \quad \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right)$$

$$n! = \int_0^\infty dx\, x^n e^{-x}, \quad \Gamma(n) = \int_0^\infty dx\, x^{n-1} e^{-x}, \quad \Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}, \quad \ln(n!) \xrightarrow[n\to\infty]{} n\ln(n) - n + \tfrac{1}{2}\ln(2\pi n)$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad I(n) = \int_0^\infty dx\, \frac{x^n}{e^x-1} = \zeta(n+1)\Gamma(n+1)$$

$$\int_0^\infty dx\, \frac{x^n e^x}{(e^x-1)^2} = n\,\zeta(n)\Gamma(n)$$

$$\mathrm{Li}_n(z) = \sum_{k=0}^{\infty} \frac{z^k}{k^n}, \quad \int_0^\infty dx\, \frac{x^{n-1}}{z^{-1}e^x \pm 1} = \mp\Gamma(n)\mathrm{Li}_n(\mp z)$$

$$-\mathrm{Li}_{3/2}(-z) \xrightarrow[z\to 0]{} z - \frac{z^2}{2^{3/2}} + \cdots$$

$$\xrightarrow[z\to\infty]{} \frac{4}{3\sqrt{\pi}}\left[(\ln z)^{3/2} + \frac{\pi^2}{8}(\ln z)^{-1/2} + \cdots\right] + \cdots$$

$$\mathrm{Li}_{3/2}(z) \xrightarrow[z\to 0]{} z + \frac{z^2}{2^{3/2}} + \cdots$$