

Write your name on the cover of the test booklet.

Do NOT panic if you can not finish all problems. Do as much as you can.

Show your work. Results without any derivation do not receive credit. Derive all results starting from the equations given. If you have memorized an equation that results from these, derive it for full credit.

(1) (a) Compute the heat capacity, $C = \partial U / \partial T$, for the Debye model in two dimensions. The dispersion relation is $\omega = v_s |\mathbf{q}|$, where v_s is the velocity of sound. There is a Debye frequency so that $\int_0^{\omega_D} g(\omega) d\omega = 2N$. (b) Approximate the high temperature limit of C .

(2) A volume V at temperature T contains N spin-1/2 fermions of mass m . (a) Compute N and the energy U as functions of the chemical potential, μ . (b) Using low temperature approximations, find $\mu(T)$ for fixed N and use this result to approximate $U(N, V, T)$ in the low temperature limit.

(3) A volume V at temperature T contains N spin-0 bosons of mass m . (a) Compute N and the energy U as functions of the chemical potential, μ . First assume there is no Bose condensate and use your results to find the critical temperature T_c . (b) Find N and U below T_c and approximate $U(N, V, T)$ in the low temperature limit.