Physics 622 WI Ø8

partition function $Z = \sum_\alpha e^{-\beta E_\alpha}$

U $\quad dU = T\,dS - p\,dV \quad U = -\frac{d}{d\beta}\ln Z$

$H = U + pV \quad dH = T\,dS + V\,dp \quad H = k_B T\left[T\left(\frac{\partial \ln Z}{\partial T}\right)_V + V\left(\frac{\partial \ln Z}{\partial V}\right)_T\right]$

$F = U - TS \quad dF = -S\,dT - p\,dV \quad F = -k_B T \ln Z$

$G = H - TS \quad dG = -S\,dT + V\,dp \quad G = k_B T\left[-\ln Z + V\left(\frac{\partial \ln Z}{\partial V}\right)_T\right]$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{U-F}{T} = k_B \ln Z + k_B T\left(\frac{\partial \ln Z}{\partial T}\right)_V$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = k_B T\left(\frac{\partial \ln Z}{\partial V}\right)_T$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = k_B T\left[2\left(\frac{\partial \ln Z}{\partial T}\right)_V + T\left(\frac{\partial^2 \ln Z}{\partial T^2}\right)_V\right]$$

density of states: $g(q)\,dq = \frac{V q^2\,dq}{2\pi^2}$ in 3d

approximation for fixed particle number $\quad Z_N \simeq \frac{(Z_1)^N}{N!}$

grand partition function $\mathcal{Z} = \sum_i e^{\beta(\mu N_i - E_i)}$

$$N = k_B T \left(\frac{\partial \ln \mathcal{Z}}{\partial \mu}\right)_\beta \; ; \; U = -\left(\frac{\partial \ln \mathcal{Z}}{\partial \beta}\right)_\mu + \mu N$$

grand potential : $\Phi_G = U - TS - \mu N = F - \mu N$

$$d\Phi_G = -S\,dT - p\,dV - N\,d\mu$$

$$\Phi_G = -k_B T \ln(\mathcal{Z})$$

$$S = -\left(\frac{\partial \Phi_G}{\partial T}\right)_{V,\mu} \qquad p = -\left(\frac{\partial \Phi_G}{\partial V}\right)_{T,\mu}$$

$$N = -\left(\frac{\partial \Phi_G}{\partial \mu}\right)_{T,V}$$

$$\mu = \frac{G}{N}, \quad \Phi_G = -pV$$

- if there is a reaction $A + B \rightleftarrows C + D$

  then $\mu_A + \mu_B = \mu_C + \mu_D$ at equilibrium

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photons : $E = \hbar\omega = \hbar q c$

phonons : $E = \hbar\omega = \hbar\omega(q)$