PHYSICS 622 **Winter 2008** **First Exam**

PUT YOUR NAME ON THE TEST BOOKLET

Do all calculations in the test booklets, organize and show all work. Results without any derivation do not receive credit. Derive all results starting from equations given. You may introduce notation to simplify calculations (e.g., abbreviations) but express all final answers in terms of quantities given in the problem. Simplify complex expressions and complete integrals (if possible) for full credit. If you make an approximation, state what it is explicitly and under what conditions it is valid. If you must assume high or low temperature, compute approximate results in both of these limits where possible (unless specifically told to make only one of these approximations in the problem).

(1) Compute the partition function, Z, and heat capacity at constant volume, C_V, for: (a) paramagnet, $E_\uparrow = -\mu B$, $E_\downarrow = \mu B$, (b) the rigid rotor, $E(j,m) = j(j+1)\epsilon$, where $j = 0, 1, 2, ...$ and $m = -j, -j+1, ..., j$, and (c) the Einstein solid, $E(n) = (n + 1/2)\epsilon$, where $n = 0, 1, 2, ...$ In each case, compute any sums and only where necessary use an approximation assuming high temperature.

(2) A volume V contains N atoms and a nonconserved number of photons at a temperature T. (a) Compute the total energy U due to both atoms and photons from their partition functions (remembering that you may need to work with $\ln(Z)$), using approximations to complete sums and integrals only where necessary and clearly stating under what condition(s) the approximations are valid. (b) Under what conditions will the photon energy dominate? If $k_B T$ must be less than $1eV$ to ignore excited states of the atoms (which you should assume you can do), try to say something about whether these conditions can be met and how, or if they can't be met why.

(3) Compute the heat capacity, $C = \partial U/\partial T$, for the Debye model in two dimensions. We still have the dispersion relation $\omega = v_s|\mathbf{q}|$, where v_s is the velocity of sound, and a Debye frequency so that $\int_0^{\omega_D} g(\omega)d\omega = 2N$. Say as much as you can about the low and high temperature limits of your result.