Today we’re not using Matlab. We’ll study the use of Lagrange multipliers for constrained minimization problems (such as problem 22.1 in the book), and we’ll start to gain some understanding of the grand partition function.

(1) To minimize a function, \( f(x, y, z) \), subject to the constraint that \( g(x, y, z) = 0 \), form the new function \( F(\lambda, x, y, z) = f(x, y, z) - \lambda g(x, y, z) \) and find its minimum with respect to \( x, y, z \) and \( \lambda \). To see how this works, find the minimum of \( \exp(-x^2 - 2y^2 + 3z^2) \) subject to the constraint that \( x^2 + y^2 = 4z \).

(2) The grand partition function is \( Z = \sum_i \exp(\beta \mu N_i - \beta E_i) \). Consider particles in a box and treat them as distinguishable. We need to sum over states with different numbers of particles, so we have a sum over \( N \). For each \( N \) we have another sum over \( n_1, n_2, \ldots, n_N \), with \( E = (n_1^2 + n_2^2 + \cdots + n_N^2)\epsilon \). Try to write the multiple sum and evaluate it. At some point you need to use a generalization of \( \sum_{n_1} \sum_{n_2} \exp(-(n_1^2 + n_2^2)\epsilon) = \sum_{n_1} \exp(-n_1^2\epsilon) \cdot \sum_{n_2} \exp(-n_2^2\epsilon) = (\sum_n \exp(-n^2\epsilon))^2 \).