

Final problem for first homework set.

Consider a 3-level system with energy levels: $E_0 = 0$, $E_1 = \Delta - \delta$, $E_2 = \Delta + \delta$, where $\delta \ll \Delta$ and both are positive. We can approximate this system by letting $\delta = 0$ and treating $E_1 = E_2 = \Delta$ with a degeneracy of 2.

$$Z_{exact} = 1 + e^{-\beta\Delta} \left(e^{\beta\delta} + e^{-\beta\delta} \right)$$

$$Z_{approximate} = 1 + 2e^{-\beta\Delta}$$

Compute the average energy $U(\beta)$ and the free energy $F(\beta)$, study the difference between the exact and approximate results, and discuss the range of β over which the approximation is 'valid.'