PHYSICS 621 **Autumn 2007** **Exam 3**

Name: Solutions - spotting errors → extra credit

Do NOT panic if you can not finish both problems. Do as much as you can.

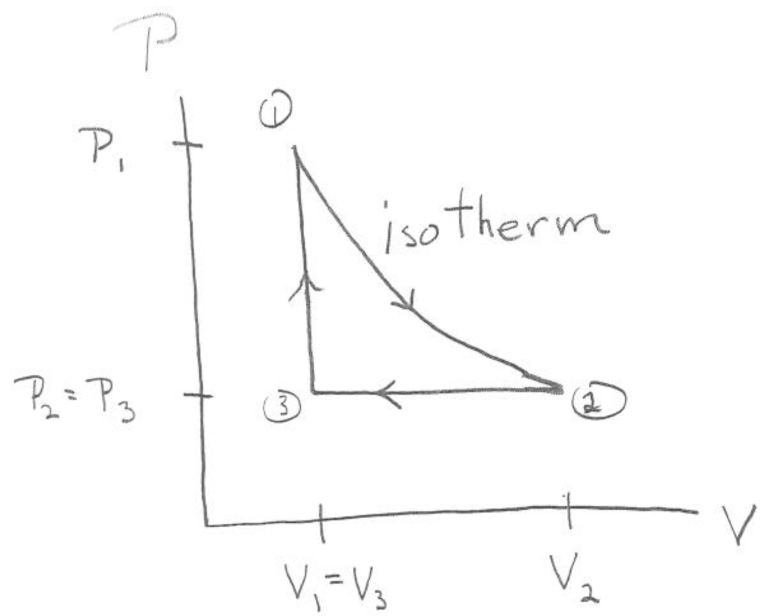
Show your work. Results without any derivation do not receive credit. Derive all results starting from the equations below. If you have memorized an equation that results from these four, derive it for full credit. Use this page for scratch work.

$\Delta U = Q + W \qquad W = -P\,dV$

$U_{\text{thermal}} = \frac{1}{2}NfkT \qquad PV = NkT \qquad PV^{\gamma} = \text{constant} \quad (\gamma = 5/3)$

$S = k\,\ln(\Omega) \qquad \frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} \qquad$ ideal gas: $\Omega = f(N)V^N U^{3N/2}$

$\ln(n!) \approx n\,\ln(n) - n \qquad \ln(1+x) = x - \frac{x^2}{2} + \cdots$



(100 pts) Given T_1, V_1 and V_2 (express all answers in terms of these three givens), compute: (a) P_1, P_2, T_2 and T_3, (b) W_{12}, W_{23}, W_{31}, (c) Q_{12}, Q_{23}, Q_{31}, and (d) $e = 1 - Q_c/Q_h$.

isotherm ① → ②

$\Delta T = 0$, $\Delta U = Q + W = 0$, $P = \frac{NkT}{V}$ } from ideal gas laws

$$\boxed{W_{12} = -\int_{V_1}^{V_2} P(V)\, dV = NkT_1 \ln\left(\frac{V_1}{V_2}\right)}$$ } negative because engine

$$\boxed{Q_{12} = -W_{12}}$$ } positive heat, part of Q_h going in to engine

$PV = NkT$ }

$$\boxed{P_1 = \frac{NkT_1}{V_1}} \quad \boxed{P_2 = \frac{NkT_1}{V_2}} \quad \boxed{T_2 = T_1}$$

② → ③ constant pressure

$$\boxed{W_{23} = -P_2 (V_3 - V_2) = NkT_1 \left(1 - \frac{V_1}{V_2}\right)}$$ positive work done to compress gas

$PV = NkT$ }

$$\boxed{P_3 = P_2 = \frac{NkT_1}{V_2}} \quad \boxed{T_3 = \frac{V_3}{V_2} T_2 = \frac{V_1}{V_2} T_1}$$ temperature drops even though work is done on gas

$$\Delta U = \tfrac{3}{2} Nk\, \Delta T = \tfrac{3}{2} Nk (T_3 - T_2) = -\tfrac{3}{2} NkT_1 \left(1 - \frac{V_1}{V_2}\right)$$

$U = \frac{3}{2}NkT$ }

$$\Rightarrow \boxed{Q_{23} = \Delta U - W = -\frac{5}{2} NkT_1 \left(1 - \frac{V_1}{V_2}\right)}$$ part of Q_c

③ → ① constant volume

$$\boxed{W_{31} = 0} \qquad \Delta U = Q = \tfrac{3}{2} Nk\, \Delta T, \quad \Delta P\, V = Nk\, \Delta T$$

$$\boxed{Q_{31} = \tfrac{3}{2} Nk (T_1 - T_3) = \tfrac{3}{2} NkT_1 \left(1 - \frac{V_1}{V_2}\right)}$$ part of Q_h

(d) efficiency $= 1 - \dfrac{Q_c}{Q_h} = 1 - \dfrac{\frac{5}{2} NkT_1 \left(1 - \frac{V_1}{V_2}\right)}{NkT_1 \ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2} NkT_1 \left(1 - \frac{V_1}{V_2}\right)}$

$$e = 1 - \frac{\frac{5}{2}\left(1 - V_1/V_2\right)}{\ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}\left(1 - V_1/V_2\right)} = 1 - \frac{\frac{5}{2} \Delta V}{V_2 \ln\left(1 + \frac{\Delta V}{V_1}\right) + \frac{3}{2} \Delta V}, \quad \Delta V = V_2 - V_1$$

Extra credit: Compute the efficiency for an engine with an identical T_1, V_1 and V_2 as in the problem above, but with an adiabatic first step rather than an isothermal first step. Which engine is more efficient?

This still gets extra credit if you can work it out. Mathematica is allowed.

adiabatic $P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow \boxed{P_2 = \left(\frac{V_1}{V_2}\right)^\gamma P_1}$

$$P_2 V_2 = NkT_2 = P_1 V_1 \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_1}\right) = NkT_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\Rightarrow \boxed{T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1}$$ gas does work & T drops

$$\Delta U = \boxed{W_{12} = \frac{3}{2}Nk\Delta T = \frac{3}{2}NkT_1\left(1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right)} \quad \boxed{Q_{12} = 0}$$

constant pressure $\boxed{P_3 = P_2 = \left(\frac{V_1}{V_2}\right)^\gamma P_1}$ $\boxed{T_3 = \frac{V_3}{V_2} T_2 = \left(\frac{V_1}{V_2}\right)^\gamma T_1}$

$$W_{23} = -P_2(V_3 - V_2) = -\left(\frac{V_1}{V_2}\right)^\gamma P_1 (V_1 - V_2) = -\left(\frac{V_1}{V_2}\right)^{\gamma-1} P_1 V_1 \left(\frac{V_1}{V_2} - 1\right)$$

$$\boxed{W_{23} = NkT_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}\left(1 - \frac{V_1}{V_2}\right)}$$

$$\Delta U_{23} = \frac{3}{2}Nk\Delta T = \frac{3}{2}Nk\left(\left(\frac{V_1}{V_2}\right)^\gamma T_1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1\right)$$

$$= -\frac{3}{2}NkT_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}\left(1 - \frac{V_1}{V_2}\right)$$

$$\boxed{Q_{23} = \Delta U_{23} - W_{23} = -\frac{5}{2}NkT_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}\left(1 - \frac{V_1}{V_2}\right)}$$ negative, so this is Q_c

constant volume $\boxed{W_{31} = 0}$ $\Delta U = Q = \frac{3}{2}Nk\Delta T$, $\Delta P\, V = Nk\Delta T$

$$\boxed{Q_{31} = \frac{3}{2}Nk(T_1 - T_3) = \frac{3}{2}NkT_1\left(1 - \left(\frac{V_1}{V_2}\right)^\gamma\right)}$$ positive, so this is Q_h

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{5}{3}\frac{\left(\frac{V_1}{V_2}\right)^{\gamma-1}\left(1 - V_1/V_2\right)}{1 - \left(\frac{V_1}{V_2}\right)^\gamma} = 1 - \frac{5}{3}\frac{\left(V_2/V_1 - 1\right)}{\left(\frac{V_2}{V_1}\right)^\gamma - 1}$$