# Physics 621 AU$\emptyset$7 Notes on 3rd Exam

- two heat engine problems
- ideal gas
- given PV diagram with steps:
  - i) isothermal
  - ii) adiabatic
  - iii) constant P
  - iv) constant V

Typical problem:

Given $P_1, V_1, V_2, V_3, V_4$, compute some of:

$P_2, P_3, P_4, T_1, T_2, T_3, T_4$ - at pts.

$W_{12}, W_{23}, W_{34}, W_{41}$ - work between pts.

$Q_{12}, Q_{23}, Q_{34}, Q_{41}$ - heat between pts

$e = 1 - \frac{Q_c}{Q_h}$ in terms of other quantities

$$dU = T\,dS - P\,dV \quad \text{(won't need } \mu\, dN\text{)}$$

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$\Delta U = W + Q$ , $W = -P\,dV$ — mechanics

$PV = NkT$ , $U = \frac{3}{2} NkT$ — ideal gas at equilibrium

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<u>isothermal</u> $\Delta T = 0$

$\Delta U = 0 \Rightarrow W = -Q$ — direct transformation of heat to work with no loss of efficiency

$$\left.\begin{aligned} P(V) &= \frac{NkT_0}{V} \\ W &= -\int_{V_i}^{V_f} dV\, \frac{NkT_0}{V} = NkT_0 \ln\left(\frac{V_i}{V_f}\right) \end{aligned}\right\} \text{isothermal work } \underline{\text{in}}$$

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<u>adiabatic</u> $Q = 0$ , $\Delta U = W$

$\Delta U = \frac{3}{2} Nk\, \Delta T$

$PV^{\gamma} = \text{constant} \Rightarrow TV^{\gamma-1} = \text{constant}$

$\gamma = 1 + \frac{2}{3} = 5/3$ note $\frac{1}{\gamma-1} = \frac{3}{2}$

$$W = -\int_{V_i}^{V_f} dV\, \frac{P_i V_i^{\gamma}}{V^{\gamma}} = P_i V_i^{\gamma}\, \frac{V_f^{-\gamma+1} - V_i^{\gamma+1}}{(\gamma-1)} \quad \text{work } \underline{\text{in}}$$

constant $V$ $\qquad W = 0 \quad , \quad \Delta U = Q$

$$\left.\begin{aligned} \Delta U &= \tfrac{3}{2} Nk\, \Delta T \\ \Delta P &= \frac{Nk}{V_0} \Delta T \end{aligned}\right\} \quad \Delta U = \tfrac{3}{2} V_0\, \Delta P$$

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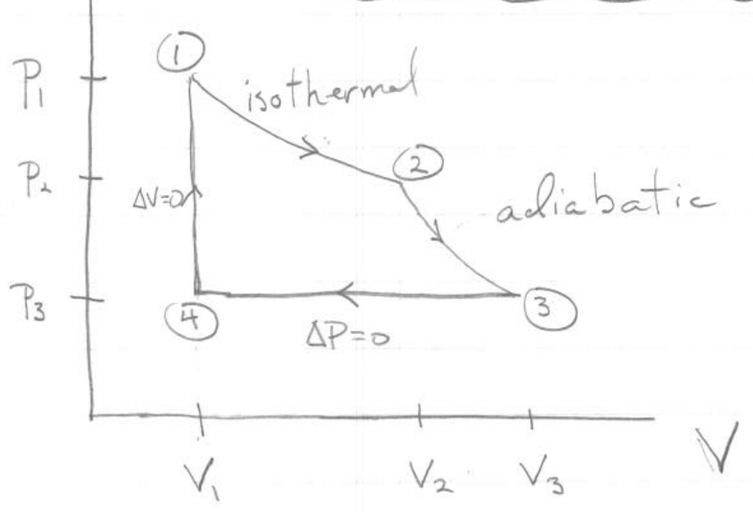
constant $P$

$$W = -P_0 \int_{V_i}^{V_f} dV = -P_0\, \Delta V$$ - work in everywhere

$$\Delta U = \tfrac{3}{2} Nk\,\Delta T = \tfrac{3}{2} P_0\, \Delta V$$

$$= -\tfrac{3}{2} W \underset{\Delta U = Q + W}{\Longrightarrow} Q = -\tfrac{5}{2} W$$ - heat in

# Example




Given $P_1, V_1, V_2, V_3$,

compute :

i) $T_1, P_2, P_3, W_{12}, W_{23}, W_{34}, W_{41}, Q_{12}, Q_{23}, Q_{34}, Q_{41}$

ii) $e = 1 - \frac{Q_c}{Q_h}$

$$T_1 = \frac{P_1V_1}{Nk}$$

$$P_2 = \frac{V_1}{V_2}P_1 \, , \quad T_2 = T_1$$

$$W_{12} = -Q_{12} = NkT_1 \ln\left(\frac{V_1}{V_2}\right) = P_1V_1\ln\left(\frac{V_1}{V_2}\right)$$

$$P_3 = \left(\frac{V_2}{V_3}\right)^{\gamma} P_2 = \left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)P_1 \, , \quad T_3 = \frac{1}{Nk}\frac{P_1V_1V_2^{\gamma-1}}{V_3^{\gamma-1}} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}T_1$$

$$W_{23} = \frac{P_2V_2^{\gamma}}{\gamma+1}\left(\frac{1}{V_3^{\gamma+1}} - \frac{1}{V_2^{\gamma+1}}\right) = \frac{1}{\gamma-1}\left[\left(\frac{V_2}{V_3}\right)^{\gamma-1} - 1\right]P_1V_1 \, , \quad Q_{23} = 0$$

$$P_4 = P_3 \, , \quad V_4 = V_1 \, , \quad T_4 = \left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)T_1$$

$$W_{34} = -P_3\left(V_4 - V_3\right) = \left(V_3 - V_1\right)\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)P_1 \, , \quad Q_{34} = -\frac{5}{2}W_{34}$$

$$W_{41} = 0 \quad , \quad Q_{41} = \frac{3}{2}Nk\left(T_1 - T_4\right) = \frac{3}{2}NkT_1\left[1-\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)\right]$$

$$\sum_{loop} \Delta U = \overset{\text{cancel}}{\cancel{W_{12}+Q_{12}}} + \frac{1}{\gamma-1}\left[\left(\frac{V_2}{V_3}\right)^{\gamma-1} - 1\right]P_1V_1$$

$$- \frac{3}{2}\left(V_3-V_1\right)\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)P_1 + \frac{3}{2}P_1V_1\left[1-\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)\right]$$

$$= \left\{\frac{3}{2}\left(\frac{V_2}{V_3}\right)^{\gamma-1} - \frac{3}{2}\left(\frac{V_2}{V_3}\right)^{\gamma-1} + \frac{3}{2}\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right) - \frac{3}{2}\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)\right]$$

$$= 0 \qquad \checkmark$$

$$e = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{\frac{5}{2}\left(V_3-V_1\right)\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)P_1}{P_1V_1\ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}P_1V_1\left[1-\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)\right]}$$

$$= 1 - \frac{\frac{5}{2}\left(\frac{V_3}{V_1}-1\right)\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)}{\ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}\left[1-\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)\right]}$$

$$\xrightarrow[V_3 = V_2]{} 1 - \frac{\frac{5}{2}\left(1 - \frac{V_1}{V_2}\right)}{\ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}\left(1-\frac{V_1}{V_2}\right)}$$