PHYSICS 621 Autumn 2007 Exam 1

Name: Solutions

Do NOT panic if you can not finish both problems. Do as much as you can.

Show your work. Results without any derivation do not receive credit. **Derive all results starting from the four equations below.** If you have memorized an equation that results from these four, derive it for full credit. Use this page for scratch work.

$$\Delta U = Q + W$$
$$U_{\text{thermal}} = \frac{1}{2} N f k T$$
$$PV = NkT$$
$$C = \frac{Q}{\Delta T}$$

heat capacity

1) What is the ~~specific~~ heat at constant pressure, C_P, for N molecules of: (a) a gas of atoms, (b) a gas of diatomic molecules with vibrational modes frozen out, (c) a gas of diatomic molecules with vibrational modes accessible, (d) a solid?

$$C_p = \left(\frac{Q}{\Delta T}\right)_P = \left(\frac{\Delta U - W}{\Delta T}\right)_P = \left(\frac{\Delta U - (-P\Delta V)}{\Delta T}\right)_P$$

$$\longrightarrow \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$U = \tfrac{1}{2} N f k T \Rightarrow \frac{\partial U}{\partial T} = \tfrac{1}{2} N f k$$ use for (a) – (d)

$$PV = NkT \Rightarrow V = \frac{Nk}{P} T \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = Nk/P$$ use for (a) – (c)

$C_p = \tfrac{1}{2} N f k + Nk$ – for ideal gases

$C_p = \tfrac{1}{2} N f k$ – For solid, where $\Delta V \sim 0$

(a) $f = 3$ (translational) $\quad C_p = \left(\frac{3}{2} + 1\right) Nk = \underline{\frac{5}{2} Nk}$ ✓

(b) $f = 3$ (trans) + 2 (rotational) $\quad C_p = \left(\frac{5}{2} + 1\right) Nk = \underline{\frac{7}{2} Nk}$ ✓

(c) $f = 3$ (trans) + 2 (rot) + 2 (vibrational)

$$C_p = \left(\frac{7}{2} + 1\right) Nk = \underline{\frac{9}{2} Nk}$$ ✓

(d) $f = 6$ (vibrational)

$$C_p = \frac{6}{2} Nk = \underline{3Nk}$$ ✓

2) A gas of N dumbbell-shaped molecules (i.e., two atoms at the ends of a rigid rod) occupies a volume V_1 at a pressure P_1. (Express answers in terms of N, V_1 and P_1.) A piston reduces the volume by half ($V_2 = V_1/2$) rapidly but "not too fast." (a) What are P_2 and T_2, the pressure and temperature after this compression? (b) How much work is done on the gas?

$f = 3$ tranlational $+ 2$ rotational

rapidly but not too fast $\Rightarrow$ adiabatic $(Q = 0)$

$$\Delta U = \cancelto{0}{Q} + W = -P\Delta V = \tfrac{1}{2}Nfk\,\Delta T$$

$$dU = -P\,dV = \tfrac{1}{2}Nfk\,dT$$

$$PV = NkT \Rightarrow P = \frac{NkT}{V}$$

$$-\frac{NkT}{V}\,dV = \tfrac{1}{2}Nfk\,dT$$

$$-\frac{dV}{V} = \frac{f}{2}\frac{dT}{T}$$

$$-\int_{V_1}^{V_2}\frac{dV}{V} = -\ln(V)\Big|_{V_1}^{V_2} = \ln\left(\frac{V_1}{V_2}\right) = \ln(2)$$

$$= \frac{f}{2}\int_{T_1}^{T_2}\frac{dT}{T} = \frac{f}{2}\ln\left(\frac{T_2}{T_1}\right) \longrightarrow \left(\frac{T_2}{T_1}\right)^{f/2} = 2$$

$$\text{or}\quad \frac{T_2}{T_1} = 2^{2/f}, \qquad \boxed{T_2 = 2^{2/5}\,T_1 \text{ or } 2^{2/5}\frac{P_1V_1}{Nk}}$$

$$P_2 = \frac{NkT_2}{V_2} = \frac{2}{V_1}Nk\,2^{2/5}\,T_1 \qquad P_2 = \frac{2^{7/5}NkT_1}{V_1}$$

$$\text{or}\quad \boxed{P_2 = 2^{7/5}\,P_1}$$

$$\int_{U_1}^{U_2} dU = \Delta U = \tfrac{1}{2}Nfk\int_{T_1}^{T_2} dT \qquad \boxed{\Delta U = \tfrac{5}{2}Nk\left[2^{2/5} - 1\right]T_1}$$