Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Circle or clearly delineate all relevant work.

1) An electron trapped in a straight wire of length $L$ can be treated as a particle in a one-dimensional infinite well confined to $0 \leq x \leq L$. (a) The electron is in the ground state. What is the probability of finding it in the region $0 \leq x \leq L/3$? (b) The electron absorbs a photon of energy $8E_1$. ($E_1$ is defined in the notes.) After absorbing the photon, what is the probability of finding it in the region $0 \leq x \leq L/3$? (c) If the electron starts at $t = 0$ in the state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\phi_2\rangle - |\phi_3\rangle)$, what is the average value of the position, $\langle x \rangle$, measured at a later time $t$?

(a) \[ P = \int_0^{L/3} dx \left| \psi(x,t) \right|^2 = \int_0^{L/3} dx \left| \frac{1}{\sqrt{L}} \sin \left( \frac{\pi x}{L} \right) e^{-i\omega t} \right|^2 = \frac{2}{L} \int_0^{L/3} dx \sin^2 \left( \frac{\pi x}{L} \right) \]

\[ = \frac{2}{L} \left\{ \frac{L}{2} - \frac{L}{4\pi} \sin \left( \frac{2\pi x}{L} \right) \right\} \left. \right|_0^{L/3} = \frac{2}{L} \left\{ \frac{L}{6} - \frac{L}{4\pi} \sin \left( \frac{2\pi x}{3} \right) \right\} \]

\[ P = \frac{1}{3} - \frac{1}{2\pi} \sin \left( \frac{\pi}{3} \right) = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} \]

Less than $\frac{1}{3}$ because this wave function peaks in the middle of the wire.

(b) $E_1 \rightarrow 9E_1$, so $n=3$ after photon is absorbed.

\[ P = \frac{2}{L} \int_0^{L/3} dx \sin^2 \left( \frac{3\pi x}{L} \right) \]

\[ = \frac{2}{L} \left\{ \frac{x}{2} - \frac{1}{12\pi} \sin \left( \frac{6\pi x}{L} \right) \right\} \left. \right|_0^{L/3} \]

\[ P = \frac{1}{3} \text{ plot } |\psi|^2 \text{ to see this} \]

(c) $|\psi(t)\rangle = e^{-i\omega t/\hbar} |\psi(0)\rangle = \frac{1}{\sqrt{2}} \left[ e^{-i\omega t} |\phi_2\rangle - e^{-i\omega t} |\phi_3\rangle \right]$;

\[ \langle x \rangle = \int_0^L dx |\psi(x,t)\rangle^* \psi(x,t) = \frac{1}{2} \int_0^L dx \left[ \phi_2^2 + \phi_3^2 - \phi_2 \phi_3 \left( e^{-i(\omega \tau) t} - e^{i(\omega \tau) t} \right) \right] \]

\[ = \frac{1}{2} \left\{ \frac{L}{2} + \frac{L}{2} - \frac{2}{L} \times \frac{L}{(3^2-2^2)\pi^2} \left[ 2 \times 3 \times 2 \times (-1+(-1)^{3/2}) \right] \times 2 \cos(\omega \tau) \right\} \]

\[ \langle x \rangle = \left[ \frac{1}{2} + \frac{48}{25\pi^2} \cos \left( 5\omega t \right) \right] L \]

- uses $\omega_n = n^2 \omega$,
- integral on board misread by many people, so I did not take pts. for minor errors.
2) A hydrogen atom is in an excited state, $n = 2$, $l = 1$, $m = 1$. (a) What is the probability that it will be found outside the Bohr radius, $r > a_0$? (b) What is the probability that it will be found inside the Bohr radius, $r < a_0$?

(a) \[ P = \int_0^\infty \int_0^{2\pi} \int_0^{\pi} r^2 \sin \theta \, dr \, d\theta \, d\phi \left| \psi_{211}(r) \right|^2 \]

\[ = \int_0^\infty r^2 \, dr \left( \frac{R_a(r)}{a_o} \right)^2 \times \left[ \sum_{\ell=0}^{\infty} \left| \psi_{\ell\ell}(\theta, \phi) \right|^2 \right] \]

\[ = 1, \text{ spherical harmonics are normalized} \]

\[ P_{r > a_o} = \frac{1}{3} \left( \frac{1}{2a_0} \right)^3 \int_0^\infty r^2 \, dr \left( \frac{r}{a_0} \right)^2 e^{-r/a_0} \]

\[ \rightarrow \frac{1}{3} \left( \frac{1}{2a_0} \right)^3 a_0^3 \int_0^\infty dx \, x^4 \, e^{-x} \]

\[ = \frac{1}{24} \left\{ \left[ 24 + 24x + 12x^2 + 4x^3 + x^4 \right] e^{-x} \right\}_{x=0}^{x=\infty} \]

\[ = \frac{1}{24} \left[ 24 + 24 + 12 + 4 + 1 \right] e^{-1} \]

\[ P_{r > a_o} = \frac{65}{24e} \approx 0.99634 \]

(b) \[ P_{r < a_o} = 1 - \frac{65}{24e} \approx 0.00366 \]

- I generously gave full credit for
- 1 - $P_{r > a_o}$ even if you got $P_{r < a_o}$ wrong.
3) What interactions are responsible for (a) $\Sigma^- \rightarrow n + \pi^-$, (b) $\Xi^* \rightarrow \Xi^0 + \pi^-$, (c) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$, (d) $p + p \rightarrow n + n + \pi^+ + \pi^+$? Justify your answers.

(a) weak because $\Delta S = 1$
(b) strong because $\Delta S = 0$ + time is $6 \times 10^{-23}$ s
(c) electromagnetic because photon is produced
(d) strong, $\Delta S = 0$ + no photons

4) What conservation laws (there may be none or more than one) are violated in the reactions: (a) $p + p \rightarrow n + \pi^+ + \pi^+$, (b) $\pi^- + n \rightarrow K^- + \Lambda^0$, (c) $\nu_\mu + n \rightarrow \mu^- + p$, (d) $p + n \rightarrow p + \pi^- + \gamma$?

(a) baryon number not conserved
(b) $\Delta S = 2$
(c) no conservation laws obviously violated, this is a weak interaction
(d) charge not conserved, baryon number not conserved

5) Supply the missing particle in the decays: (a) $K^- \rightarrow \pi^0 + e^- + \bar{\nu}$, (b) $\eta \rightarrow \pi^+ + \pi^- + \pi^0$, (c) $K^0 \rightarrow \pi^0 + \pi^0$? Justify your answers.

(a) $\bar{\nu}$ needed to conserve le, electron lepton
(b) not obvious another particle is required, but $\pi^0$ works if this is a strong interaction. All other mesons are too massive.
(c) $\pi^0$ again; weak interaction since $\Delta S = 1$, but no single lepton can appear + no other meson is light enough

Most of this problem is identical to the last homework, so I graded “harshly”.