Similarity Renormalization Group and Evolution of Many-Body Forces

E.D. Jurgenson

Department of Physics, The Ohio State University

TRIUMF National Laboratory - Vancouver, BC - February 10, 2009

Work supported by NSF and UNEDF/SciDAC (DOE)
Outline

- Overview of Similarity Renormalization Group (SRG)
- Decoupling between high- and low-energy degrees of freedom
- Evolving three-nucleon forces (3NFs in 3D!!)
- One-Dimensional Model
- Future Work and Conclusions
Renormalization Group $\implies$ focus on relevant dof’s
Which picture should I use?
Fourier transform in partial waves (Bessel transform)

\[ V_{L=0}(k, k') = \int d^3r j_0(kr)V(r)j_0(k'r) = \langle k | V_{L=0} | k' \rangle \]

- Repulsive core \( \rightarrow \) big high-k (\( \geq 2 \text{ fm}^{-1} \)) components
- EFTs are softer - but still have high-k components
Although momentum is continuous in principle, in practice represented as discrete (gaussian quadrature) grid:

\[
\langle k | V | k' \rangle + \sum_{k''} \frac{\langle k | V | k' \rangle \langle k' | V | k \rangle}{(k^2 - k''^2)/m} + \cdots \Rightarrow V_{ii} + \sum_{i} V_{ij} V_{ji} \frac{1}{(k_i^2 - k_j^2)/m} + \cdots
\]

100 \times 100 Resolution is sufficient for many significant figures
Try a Low-Pass Filter

- Start with a potential (AV18 - $^1S_0$)
- Cut at $\Lambda$ (2.2 fm$^{-1}$)
- Compute observables ($\delta_0(E)$)
- Compare to uncut
Try a Low-Pass Filter

- Start with a potential (AV18 - $^1S_0$)
- Cut at $\Lambda$ (2.2 fm$^{-1}$)
- Compute observables ($\delta_0(E)$)
- Compare to uncut
Try a Low-Pass Filter

- Start with a potential \((\text{AV18} - {}^1S_0)\)
- Cut at \(\Lambda\) (2.2 fm\(^{-1}\))
- Compute observables \((\delta_0(E))\)
- Compare to uncut
Try a Low-Pass Filter

1. Start with a potential $(AV18 - ^1S_0)$
2. Cut at $\Lambda$ (2.2 fm$^{-1}$)
3. Compute observables ($\delta_0(E)$)
4. Compare to uncut
Basic problem: high and low are coupled!

Perturbation theory for \( \tan(\delta_0) \)

\[
\langle k | V | k \rangle + \sum_{k'} \frac{\langle k | V | k' \rangle \langle k' | V | k \rangle}{(k^2 - k'^2)/m} + \ldots
\]

Can’t just change high-momentum elements (intermediate virtual states)

Absorb high-energy effects into low-energy physics ⇒ “Renormalization Group” (“flow equations”)

Unitary transformation:

\[
E_n = (\langle \psi_n | U^\dagger ) U H U^\dagger (U | \psi_n \rangle)
\]
What is the Similarity Renormalization Group (SRG)?


\[ H_s = U(s) H U^\dagger(s) \equiv T_{\text{rel}} + V_s \]

\[ \frac{dH_s}{ds} = [\eta(s), H_s] \quad \text{where} \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s) \]

\[ \eta(s) = [T_{\text{rel}}, H_s] \implies \frac{dH_s}{ds} = [[[T_{\text{rel}}, H_s], H_s]] \]

Projected onto partial-wave momentum space:

\[ \frac{dV_s(k, k')}{ds} = -(\epsilon_k - \epsilon_{k'})^2 V_s(k, k') \]

\[ + \frac{2}{\pi} \int_0^\infty q^2 dq \left( \epsilon_k + \epsilon_{k'} - 2\epsilon_q \right) V_s(k, q) V_s(q, k') \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\[ ^1S_0 \quad \lambda = 20.0 \text{ fm}^{-1} \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s)H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\[^1S_0 \quad \lambda = 15.0 \text{ fm}^{-1}\]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U(s)^\dagger \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\( ^1S_0 \quad \lambda = 12.0 \text{ fm}^{-1} \)

\[ k' \text{ (fm}^{-1}) \]

\[ k \text{ (fm}^{-1}) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \quad \Rightarrow \quad \frac{dH_s}{ds} = [[T_{\text{rel}}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{\text{rel}}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\(^1S_0 \quad \lambda = 6.0 \text{ fm}^{-1}\)

\(^1S_0 \quad \lambda = 8.0 \text{ fm}^{-1}\)
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \Rightarrow \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \quad \Rightarrow \quad \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\[ ^1S_0 \quad \lambda = 5.0 \text{ fm}^{-1} \]
What is the Similarity Renormalization Group (SRG)?

$$H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4)$$

$$^1S_0 \quad \lambda = 4.0 \text{ fm}^{-1}$$

E.D. Jurgenson  
SRG and 3NF
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \Rightarrow \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{\text{rel}}, H_s], H_s] \quad (\lambda = 1/s^4) \]

\[ ^1S_0 \quad \lambda = 2.8 \text{ fm}^{-1} \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s)H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s)H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
What is the Similarity Renormalization Group (SRG)?

\[ H_s = U(s) H U^\dagger(s) \implies \frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \quad (\lambda = 1/s^4) \]
Unitary Transformations $\Rightarrow$ Preserve Observables

\[ \delta \text{ [deg]} \]

\begin{align*}
\text{\textsuperscript{1}S}_0 & \quad \text{\textsuperscript{1}P}_1 & \quad \text{\textsuperscript{3}P}_0 \\
\text{\textsuperscript{3}P}_1 & \quad \text{\textsuperscript{3}F}_3 & \quad \text{\textsuperscript{3}G}_4
\end{align*}

\[ E_{\text{lab}} \text{ [MeV]} \]

E.D. Jurgenson
SRG and 3NF
Now Low-Pass Filters Work!

- Phase shifts with $V_s(k, k') = 0$ for $k, k' > k_{\text{max}}$

![Graphs showing phase shifts for different states](image-url)
Testing Decoupling Quantitatively

[Bogner, Furnstahl, Perry, EDJ - arXiv:0711.4252]

$\Lambda$

$4.0$

$3.4$

$2.5$

$\lambda = 5.0 \text{ fm}^{-1}$

$\lambda = 3.0 \text{ fm}^{-1}$

$\lambda = 2.0 \text{ fm}^{-1}$

$\lambda = 1.0 \text{ fm}^{-1}$

Tool for Study

1. run SRG to $\lambda$

2. set tail to zero

   $V_{s,\Lambda} = e^{-\left(\frac{k^2}{\Lambda^2}\right)^n} V_s e^{-\left(\frac{k'_2}{\Lambda^2}\right)^n}$

   $n = 4, 8, 12, \ldots$

3. relative errors

$^1S_0$ Partial Wave, N$^3$LO (500 MeV) E/M
Decoupling above $\lambda$

$^1S_0$ \text{ N}^3\text{LO}

$\lambda = 2.0 \text{ fm}^{-1}$

\[ V_{\lambda,\Lambda} = e^{-\frac{(k^2/\Lambda^2)^n}{2}} V_{\lambda} e^{-\left(k^2/\Lambda^2\right)^n} \]

- Decoupling clean and universal for all observables!
Decoupling clean and universal for all observables!

\[
\frac{dH_s}{ds} = [[G_s, H_s], H_s]
\]

\[
H_\infty = \begin{pmatrix}
PH_\infty P & 0 \\
0 & QH_\infty Q
\end{pmatrix} \implies G_s = \begin{pmatrix}
PH_s P & 0 \\
0 & QH_s Q
\end{pmatrix}
\]
Many-Body Forces

- Why do we need many-body forces?
  - 3NFs arise from eliminating dof’s
  - Omitting 3NFs leads to model dependence (Tjon line)
  - 3NF contributions saturate nuclear matter
  - Many-body methods must deal with them (e.g. MFD, CC, ...)

- SRG will induce many-body forces!

\[ \frac{dV}{ds} = \left[ \sum a^\dagger a, \sum a^\dagger a^\dagger aa, \sum a^\dagger a^\dagger aa \right] \]

\[ = \ldots + \sum a^\dagger a^\dagger a^\dagger aaa + \ldots \]

- Stop evolution if induced become too unnatural
- RG flows with SRG extend consistently to many-body spaces
- Recent progress: 3NF evolved!!!
Current Realistic NCSM Calculations

- Triton calculations from P. Navratil (arXiv:0707.4680)

\[ E_{[\text{MeV}]} \]

3\(^{\text{H}}\)

- "eff" \(\equiv\) Lee-Suzuki: Generated in large space and cut down with LS
- 3NF is N2LO
- \(N_{\text{max}} = 40\) converged to within 5keV

- 3NF parameters \(c_E\) and \(c_D\) are fit to two observables
Evolving NN Forces in NCSM $A=3$ space

MATLAB evolved $N_{\text{max}} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar\omega = 28$ is optimal for initial interaction, $\hbar\omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence
MATLAB evolved $N_{max} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar \omega = 28$ is optimal for initial interaction, $\hbar \omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence
Evolving NN Forces in NCSM $A=3$ space

MATLAB evolved $N_{\text{max}} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar \omega = 28$ is optimal for initial interaction, $\hbar \omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence

E.D. Jurgenson  SRG and 3NF
Evolving NN Forces in NCSM $A=3$ space

MATLAB evolved $N_{\text{max}} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar \omega = 28$ is optimal for initial interaction, $\hbar \omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence
Evolving NN Forces in NCSM A=3 space

MATLAB evolved $N_{\text{max}} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar \omega = 28$ is optimal for initial interaction, $\hbar \omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence
Evolving NN Forces in NCSM A=3 space

MATLAB evolved $N_{max} = 30$ in only $\approx$ half a day

- Unitary evolution of initial NN-only forces!
- $\hbar\omega = 28$ is optimal for initial interaction, $\hbar\omega = 20$ for $\lambda = 2$
- Need to study the trade-off in convergence
MATLAB evolved $N_{max} = 30$ in only $\approx$ half a day

- Same plots but now including an initial 3NF from N2LO
- Collaboration with P. Navratil - up next: size of induced 4NF in $^4$He calculations
Start with a One-Dimensional Model

- 1-D model: \( V^{(2)}(x) = \frac{V_1}{\sigma_1 \sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2 \sqrt{\pi}} e^{-x^2/\sigma_2^2} \)


How do we handle many-body forces? 

→ use a discrete basis to avoid “dangerous” delta functions

- R. J. Furnstahl and EDJ - [arXiv:0809.4199]
• HO wavefunction examples $\psi_n(k)$ with $\hbar \omega = 4$
• resulting truncated delta function
  $\tilde{\delta}(k - k') = \sum_{n=0}^{N_{\text{max}}} |\psi_n(k)\rangle\langle\psi_n(k')|$
• tradeoff between small $\hbar \omega$ resolution and large $\hbar \omega$ scope
• bigger $N_{\text{max}} \rightarrow$ flatter in $\hbar \omega$
• optimal $\hbar \omega$ will shift with SRG evolution
Embedding: initial potential

- Symmetrized Jacobi Oscillator Basis (here: Bosons)
- R. J. Furnstahl and EDJ - [arXiv:0809.4199]

$V(p, p') \rightarrow V(N_2, N'_2) \rightarrow V(N_3, N'_3)$

- Diagonalize symmetrizer $\Rightarrow \langle N_A | N_{A-1}; n_{A-1} \rangle$; use recursively
- 3D: Use Navratil et al. technology for NCSM
- Embedding is everything, SRG coding is trivial
Embedding: evolved potential - $\lambda = 2$

- Symmetrized Jacobi Oscillator Basis (here: Bosons)
- R. J. Furnstahl and EDJ - [arXiv:0809.4199]

\[ V(p, p') \rightarrow V(N_2, N_2') \rightarrow V(N_3, N_3') \]

- Diagonalize symmetrizer $\Rightarrow \langle N_A||N_{A-1}; n_{A-1}\rangle$; use recursively
- 3D: Use Navratil et al. technology for NCSM
- Embedding is everything, SRG coding is trivial
Some many-body examples

Legend: Embedding, Evolving, BE calculation, Initial 3NF

- **A=3 (2N only):**

  $$V_{osc}^{(2)} \overset{SRG}{\rightarrow} V_{\lambda,osc}^{(2)} \overset{embed}{\rightarrow} V_{\lambda,3Nosc}^{(2)} \overset{diag}{\rightarrow} BE_{3}^{(2Nonly)}$$

- **A=4 (2N only):**

  $$V_{osc}^{(2)} \overset{SRG}{\rightarrow} V_{\lambda,osc}^{(2)} \overset{embed}{\rightarrow} V_{\lambda,3Nosc}^{(2)} \overset{embed}{\rightarrow} V_{\lambda,4Nosc}^{(2)} \overset{diag}{\rightarrow} BE_{4}^{(2Nonly)}$$

- **A=4 (2N+3N only):**

  $$V_{osc}^{(2)} \overset{embed}{\rightarrow} V_{3Nosc}^{(2)} \overset{SRG}{\rightarrow} V_{\lambda,3Nosc}^{(2+3)} \overset{embed}{\rightarrow} V_{\lambda,4Nosc}^{(2+3)} \overset{diag}{\rightarrow} BE_{4}^{(2N+3Nonly)}$$

  $$\overset{3NF}{\longrightarrow} + V_{3Nosc}^{(3init)} \cdots$$
Induced Many-Body Forces are Small - A=3

\[ V^{(2)}(x) = \frac{V_1}{\sigma_1 \sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2 \sqrt{\pi}} e^{-x^2/\sigma_2^2} \]

- Basis independent: same evolution in momentum or HO basis
- Black: Same evolution pattern for 2-body only as 3D NN-only
Induced Many-Body Forces are Small - \( A = 3 \)

\[
V^{(2)}(x) = \frac{V_1}{\sigma_1 \sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2 \sqrt{\pi}} e^{-x^2/\sigma_2^2}
\]

- Basis independent: same evolution in momentum or HO basis
- Black: Same evolution pattern for 2-body only as 3D NN-only
- Red: Three-body forces induced - Unitary!
Induced Many-Body Forces are Small - $A=4$

\[ V^{(2)}(\lambda=\infty) = V_\alpha \]

\[ V^{(3)}(p, q, p', q') = c_E e^{-((p'^2+q'^2)/\Lambda^2)^n} e^{-((p^2+q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4) \]
Induced Many-Body Forces are Small - $A=4$

$A = 4 \quad N_{\text{max}} = 28$

$V^{(2)}(\lambda=\infty) = V_\alpha$

$c_E = -0.05$

$c_E = 0.00$

$c_E = 0.05$

$V^{(3)}(p, q, p', q') = c_E e^{-((p'^2 + q'^2)/\Lambda^2)^n} e^{-((p^2 + q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4)$
Induced Many-Body Forces are Small - $A=5$

- Five-body force is negligible
- Hierarchy of induced many-body forces
\[
\frac{d}{d\lambda} \langle \psi_{\lambda}^{(3)} | V_{\lambda}^{(3)} | \psi_{\lambda}^{(3)} \rangle = \langle \psi_{\lambda}^{(3)} | [\overline{V}_{\lambda}^{(2)}, V_{\lambda}^{(2)}]_c - [\overline{V}_{\lambda}^{(3)}, V_{\lambda}^{(3)}] \rangle_{\psi_{\lambda}^{(3)}}
\]

- **Majority evolution dominated by** \([\overline{V}^{(2)}, V^{(2)}]\), \((\overline{V} \equiv [T, V])\)
- **Hierarchy of contributions**

\[\begin{array}{c}
V^{(3)} \text{ analysis} \\
\end{array}\]
\[ \frac{d}{d\lambda} \langle \psi^{(4)}_\lambda | V^{(4)}_\lambda | \psi^{(4)}_\lambda \rangle = \langle \psi^{(4)}_\lambda | [V^{(2)}_\lambda, V^{(3)}_\lambda]_c + [V^{(3)}_\lambda, V^{(2)}_\lambda]_c + [V^{(3)}_\lambda, V^{(3)}_\lambda]_c - [V^{(4)}_\lambda, V^{(4)}_\lambda] | \psi^{(4)}_\lambda \rangle \]

**Graphical Analysis**

- **Ground-State Energy**
  - \( A = 4 \), \( N_{\text{max}} = 28 \)
  - \( V_\alpha \)
  - \( c_E = 0 \)

- **Ground-State Expectation Value**
  - \( A = 4 \), \( N_{\text{max}} = 28 \)
  - \( V_\alpha \)
  - \( c_E = 0 \)
  - No \([V^{(2)}_\lambda, V^{(2)}_\lambda]\): \( \times \times \)
  - \( \therefore \) Induced 4-body is small - Hierarchy persists
Evolve in two-particle oscillator space → fit 3-body parameters to missing energy.

One term \( V^{(3)} = C e^{-[(k^2 + k'^2)/\Lambda^2]^n} \) reduces \( \lambda \) dependence to the 80-90\% level.

Future work: add a second, short distance 3NF term with a gradient correction to test systematic reduction.
Operator Evolution

- $H_s = U_s^\dagger H_0 U_s \implies U_s = \sum_i |\psi_i(0)\rangle \langle \psi_i(s)|$
- Here unevolved operator ($a^\dagger a$) with evolved wavefuntions

More of this to come from E. R. Anderson
Decoupling in the Oscillator Basis

⇒ Evolve with $T_{rel}$ and cut off to study decoupling

SRG space: 2-body only
- Decoupling not straightforward with $T_{rel}$ SRG
- Decoupling improves until some $\lambda$ and then degrades
- What about other SRG generators?

E.D. Jurgenson  SRG and 3NF
Using other SRG Generators

- Matrices in NCSM basis for $T_{\text{rel}}$ and $V$

In this basis $T_{\text{rel}}$ will not drive to diagonal

- Harmonic Oscillator Hamiltonian ($H_{\text{ho}} = T_{\text{rel}} + V_{\text{ho}}$) is diagonal in this basis
Using $G = H_{ho}$ improves convergence dramatically.

- Compare $T_{rel}$ on the left with $H_{ho}$ on the right.
- Work in progress: Spurious bound states contaminate evolution with $H_{ho}$ → need further investigation.
Recap

- SRG Decouples high- and low-energy DOF
- SRG is very flexible and powerful - can use many shapes to evolve potentials
- One-Dimensional model gives proof-of-principle of many-body hierarchy
- One-Dimensional model provides toolbox to gain intuition quickly - everything is directly applicable to 3D NCSM
- First results now coming in 3NF evolution in the NCSM basis!
Future Work

- One-Dimensional model leaves many opportunities for quick exploration
  - Operator evolution
  - SRG generators ($H_{ho}$, $H_{BD}$, $H_{D}$)
  - Basis issues
  - Fitting procedures
- All of these can be started in 3D now
- Door is opening quickly to other areas (MFD, CC, UNEDF collaboration)
Unitary transformations: [PRC 75:(2007)061001
arXiv:nucl-th/0611045]

\[ H_s = U(s) H U^\dagger(s) \equiv T_{rel} + V_s \]

\[ \frac{dH_s}{ds} = [\eta(s), H_s] \quad \text{where} \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s) \]

\[ \eta(s) = [T_D, H_s] \quad \implies \quad \frac{dH_s}{ds} = [[T_D, H_s], H_s] \]

Projected onto partial-wave momentum space:

\[ \frac{dV_s(k, k')}{ds} = -(\epsilon_k - \epsilon_{k'})^2 V_s(k, k') \]

\[ + \frac{2}{\pi} \int_0^\infty q^2 dq \, (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) \, V_s(k, q) \, V_s(q, k') \]
\[
\frac{dV_s(k, k')}{ds} = -(\epsilon_k - \epsilon_{k'})^2 V_s(k, k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_s(k, q) V_s(q, k')
\]

Off-diagonal elements
\[
\Rightarrow V_s(k, k') \propto V_{NN}(k, k') e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2}
\]

Relevant physics flows to low momentum elements
Phase Shifts: Decoupled above $\lambda$ - vary $\lambda$

- Relevant physics flows to low momentum → Decoupling!
Phase Shifts: Decoupled above $\lambda - \text{vary } n$

Relevant physics flows to low momentum $\rightarrow$ Decoupling!
Deuteron Observables

- Binding Energy
- Quadrupole Moment
- RMS radius

\[ \lambda = 2.0 \text{ fm}^{-1} \]
dependence on $N_{\text{max}}$

\[ \frac{\Delta E}{E} \text{ vs } \Lambda \text{ [fm}^{-1}] \]

\( ^4\text{He} \)

\( \lambda = 2.0 \text{ fm}^{-1} \)

- $N_{\text{max}} = 4$
- $N_{\text{max}} = 8$
- $N_{\text{max}} = 12$

\[ \frac{1}{\Lambda^{16}} \]

dependence on $\lambda$

\[ \text{Energy [MeV]} \text{ vs } \Lambda \text{ [fm}^{-1}] \]

\( ^4\text{He} \)

$N_{\text{max}} = 12$

- $\lambda = 3$
- $\lambda = 2$
- $\lambda = 1.5$

- SRG improves convergence with basis size in NCSM
- NN-only $\rightarrow$ different $^4\text{He}$ Binding Energies
Li Energy using No Core Shell Model

dependence on $\lambda$

dependence on $n$

$^6\text{Li}$

$N_{\text{max}} = 8$

SRG improves convergence with basis size in NCSM

NN-only $\iff$ different $^6\text{Li}$ Binding Energies

E.D. Jurgenson

SRG and 3NF
See edj et al.: arXiv:0801.1098

\[
\text{Goal} \rightarrow H_\infty = \begin{pmatrix}
PH_\infty P & 0 \\
0 & QH_\infty Q
\end{pmatrix}
\]

\[
\text{SRG} \rightarrow \frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]
\]

\[
\text{sharp} \rightarrow G_s = \begin{pmatrix}
PH_s P & 0 \\
0 & QH_s Q
\end{pmatrix}
\]

\[
\text{smooth} \rightarrow G_s = fH_s f + (1 - f)H_s (1 - f)
\]

\[
f(k) = e^{-(k^2/\Lambda_{BD}^2)^n}
\]
Block-Diagonal SRG - Sharp

3S1, kvbn = 10, λ = 0.5 fm⁻¹, Z₂lim = 0.5

3S1, kvbn = 10, λ = 1.0 fm⁻¹, Z₂lim = 0.5

3S1, kvbn = 10, λ = 1.0 fm⁻¹, Z₂lim = 0.5

3S1, kvbn = 10, λ = 2.0 fm⁻¹, Z₂lim = 0.5

3S1, kvbn = 10, λ = 3.0 fm⁻¹, Z₂lim = 0.5

3S1, kvbn = 10, λ = 5.0 fm⁻¹, Z₂lim = 0.5
Block-Diagonal SRG - Smooth (n=4)

3S1, kvnn = 10, \( \lambda = 3.0 \text{ fm}^{-1} \), \( Z_{\text{lim}} = 0.5 \)

3S1, kvnn = 10, \( \lambda = 2.0 \text{ fm}^{-1} \), \( Z_{\text{lim}} = 0.5 \)

3S1, kvnn = 10, \( \lambda = 1.5 \text{ fm}^{-1} \), \( Z_{\text{lim}} = 0.5 \)

3S1, kvnn = 10, \( \lambda = 1.1 \text{ fm}^{-1} \), \( Z_{\text{lim}} = 0.5 \)

3S1, kvnn = 10, \( \lambda = 1.0 \text{ fm}^{-1} \), \( Z_{\text{lim}} = 0.5 \)
UNEDF Interconnections for Ab Initio Functionals

Ab Initio WF Methods
- CC: UT/ORNL (Dean, Hagen, Papenbrock)
- NCFC: ISU (Maris, Vary)
- LLNL (Navratiil)

Wider range of nuclei
- Full 3NF
- Ab Initio densities
- External potentials

Tests of DME: energies, densities with same H
- Vary 3NF, external potential parameters
- Cutoff dependence as diagnostic

Ab Initio Functional + Nuclear Matter
- OSU (Drut, Furnstahl, Platter)
- MSU (Bogner, Gebremariam)
- (also Saclay, TRIUMF)

Tests of nuclear matter:
- new fits, self-energies, ...
- Improved 3NF for DME
- Generalized DME
- DFT from OPM

Long-range pion contributions from NN and NNN DME
- plus fit residual Skyrme in HFB code

DFT Applications
- UT/ORNL (Schunck, Stoitsov)
- UW (Bertsch)
- Saclay (Duguet, Lesinski, ...)

Systematics along isotope chains
- Tests: spin-orbit splittings, time-odd terms, ...
- Non-empirical pairing functional

Interactions
- Chiral EFT
- Bonn/Julich (Epelbaum, Nogga)
- Salamanca/Idaho (Entem, Machleidt)

Vlowk/SRG
- OSU, MSU
- TRIUMF (Schwenk)

N3LO 3NF
- Explicit Delta’s
- New 3NF fits
- SRG 3NF evolution

Participant color key:
- UNEDF
- International collaborator
- Outside UNEDF
UNEDF Interconnections for Ab Initio Functionals

Ab Initio WF Methods
- CC: UT/ORNL (Dean, Hagen, Papenbrock)
- NCFC: ISU (Maris, Vary)
- LLNL (Navratil)

Interactions
- Chiral EFT
- Bonn/Julich (Epelbaum, Nogga)
- Salamanca/Idaho (Entem, Machleidt)
- Vlowk/SRG
- OSU, MSU
- TRIUMF (Schwenk)

Wider range of nuclei
- Full 3NF
- Ab Initio densities
- External potentials

Tests of DME: energies, densities with same H
- Vary 3NF, external potential parameters
- Cutoff dependence as diagnostic

Ab Initio Functional + Nuclear Matter
- OSU (Drut, Furnstahl, Platter)
- MSU (Bogner, Gebremariam)
- (also Saclay, TRIUMF)

Tests of nuclear matter:
- new fits, self-energies, ...
- Improved 3NF for DME
- Generalized DME
- DFT from OPM

DFT Applications
- UT/ORNL (Schunck, Stoitsov)
- UW (Bertsch)
- Saclay (Duguet, Lesinski, ...)

Systematics along isotope chains
- Tests: spin-orbit splittings, time-odd terms, ...
- Non-empirical pairing functional

Long-range pion contributions from NN and NNN DME
- plus fit residual Skyrme in HFB code

Participant color key:
- UNEDF
- International collaborator
- Outside UNEDF

N3LO 3NF
- Explicit Delta’s
- New 3NF fits
- SRG 3NF evolution

SRG and 3NF