The Renormalization Group Approach to the Nucleon-Nucleon Interaction

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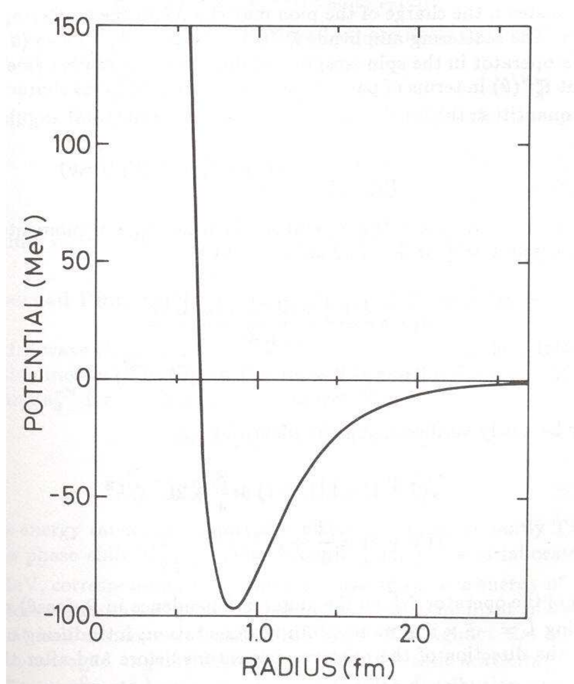
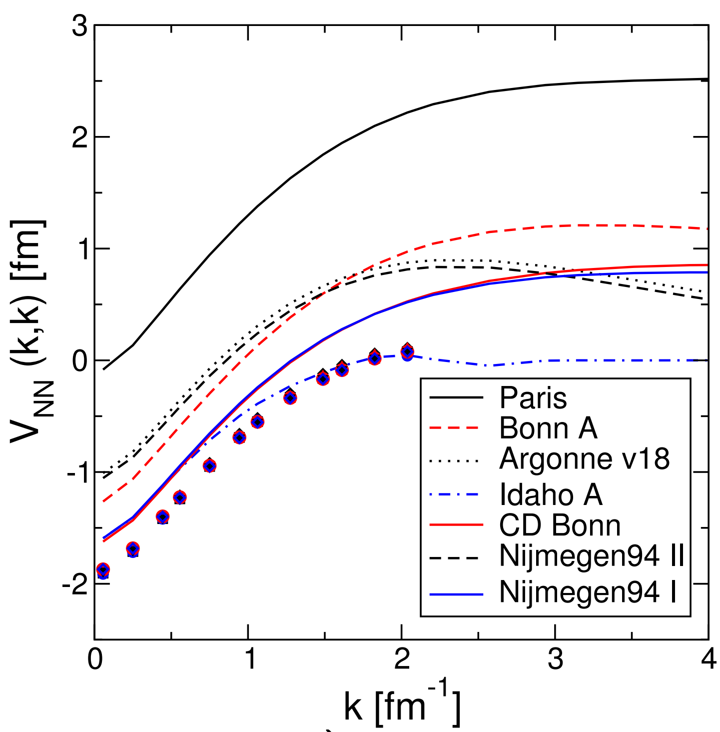
MOTIVATION

- Schematic of NN potential:



- Realistic Nucleon-Nucleon potential models



(Bogner, Kuo and Schwenk)

- In Momentum space, Lippmann-Schwinger equation for the T matrix (in units $\frac{\hbar^2}{2\mu} = \frac{\hbar^2}{m} = 1$ and assuming $\mu = m/2$),

$$\langle \vec{k'} | T^{\pm}(k^2) | \vec{k} \rangle = \langle \vec{k'} | V | \vec{k} \rangle + \int d^3q \frac{\langle \vec{k'} | V | \vec{q} \rangle \langle \vec{q} | T^{\pm}(k^2) | \vec{k} \rangle}{k^2 - q^2 \pm i\epsilon}$$

- For the l^{th} partial wave

$$\langle k' | T_l(k^2) | k \rangle = \langle k' | V_l | k \rangle + \frac{2}{\pi} \mathcal{P} \int_0^{\infty} q^2 dq \frac{\langle k' | V_l | q \rangle \langle q | T_l(k^2) | k \rangle}{k^2 - q^2}$$

- Physical Observables - Phase shifts are related to the on-shell T matrix element:

$$\tan(\delta_l) = -T_l(k, k; k^2) k$$

- Enter R.G.:
  - Introduce a cut-off Λ on intermediate state momenta.
  - $V_l \to V^{\Lambda}$.

- T matrix equation now reads,

$$T_l(k', k; k^2) = V^{\Lambda}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^{\Lambda} q^2 dq \frac{V^{\Lambda}(k', q) T_l(q, k; k^2)}{k^2 - q^2}$$

- Physical Observables are independent of the cut-off, therefore:

$$\frac{dT(k',k;k^2)}{d\Lambda} = 0.$$

R.G. EQUATION

- Differential Equation for V^{Λ}: with an appropriate regulator

$$\frac{dV^{\Lambda}(k',k)}{d\Lambda} = \frac{2}{\pi}\frac{V^{\Lambda}(k',\Lambda)T(\Lambda,k;\Lambda^2)}{1-\frac{k^2}{(\Lambda+\epsilon)^2}}$$

$$T(\Lambda,k;\Lambda^2) = V^{\Lambda}(\Lambda,k) + \frac{2}{\pi}P\int_0^{\Lambda}\frac{q^2 dq T(\Lambda,q)V^{\Lambda}(q,k)}{(\Lambda+\epsilon)^2-q^2}$$

- Solve R.G. equation with initial condition: $V^{\Lambda_0}(k',k) = V_{bare}(k',k)$ and $V_{bare}(\Lambda_0,\Lambda_0) \approx 0$

- Separable Potentials: $V_{bare}(k',k) = L(k')R(k)$
- T matrix has the following form:

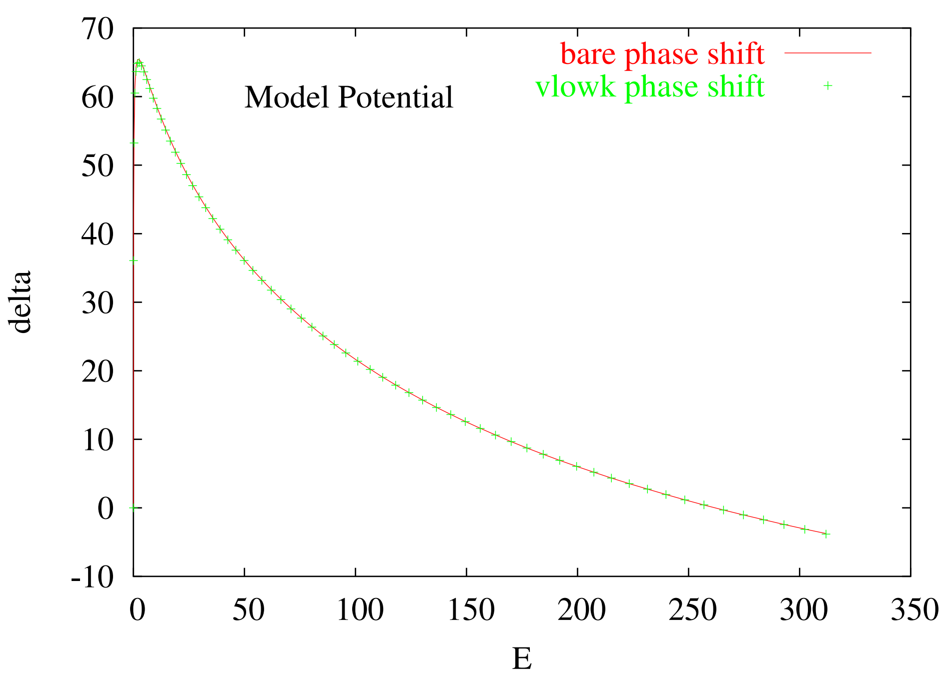
$$T(k',k;k^2) = \frac{L(k')R(k)}{1 + \frac{2}{\pi} P \int_0^{\Lambda} q^2 \, dq \frac{L(q)R(q)}{k^2 - q^2}}$$

- Model Potential:

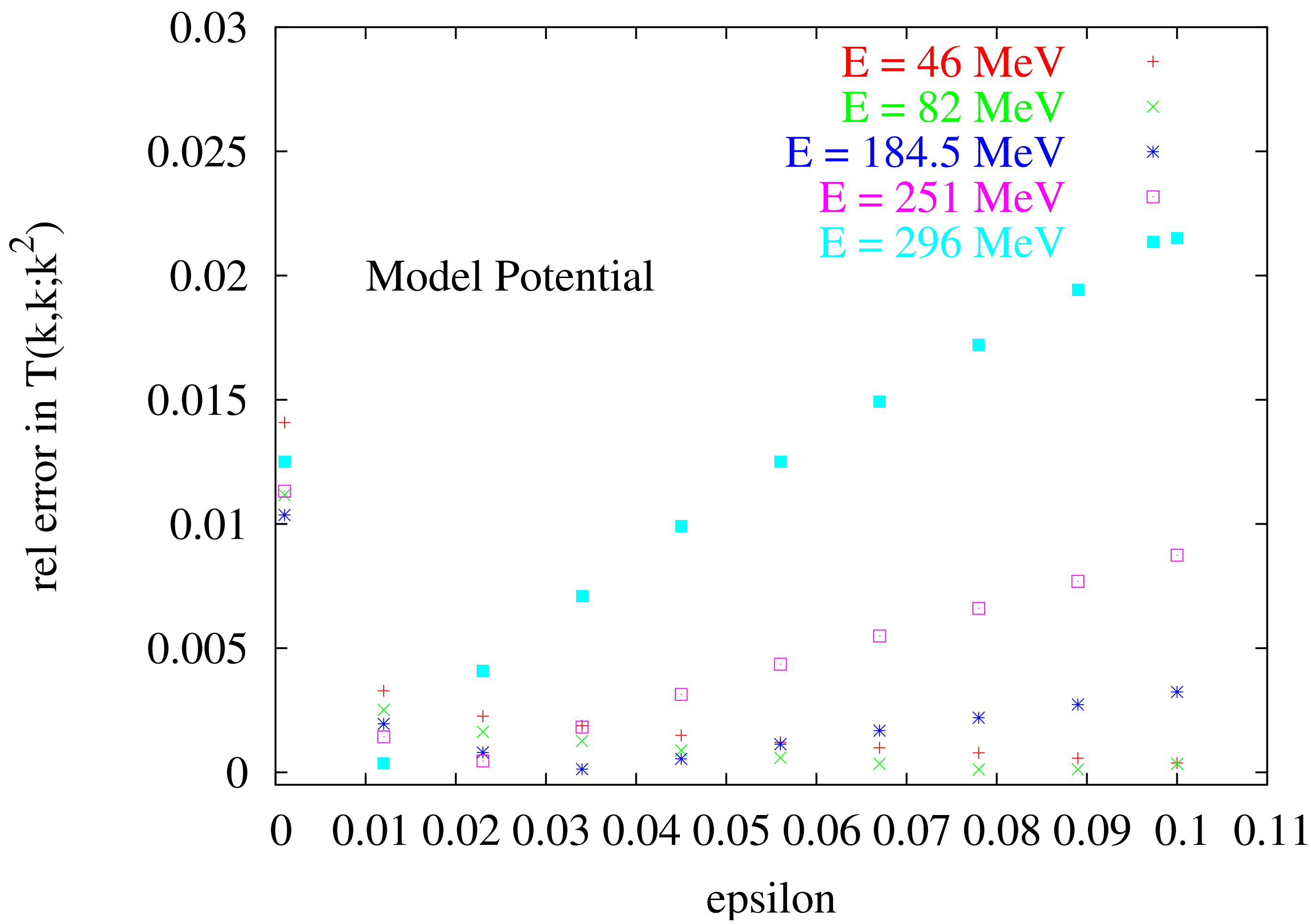
$$V_{bare}(k',k) = \frac{g}{(k'^2+m^2)^n}\left(\frac{-g}{(k^2+m^2)^n} + \frac{\alpha g (k^2+m^2)^n}{(k^2+\eta^2 m^2)^{2n}}\right)$$

$$V_{bare}(k,k) = \frac{-g^2}{(k^2+m^2)^{2n}} + \frac{\alpha g^2}{(k^2+\eta^2 m^2)^{2n}}$$

- Phase Shifts for model potential:



- Relative Error in $T(k, k; k^2)$

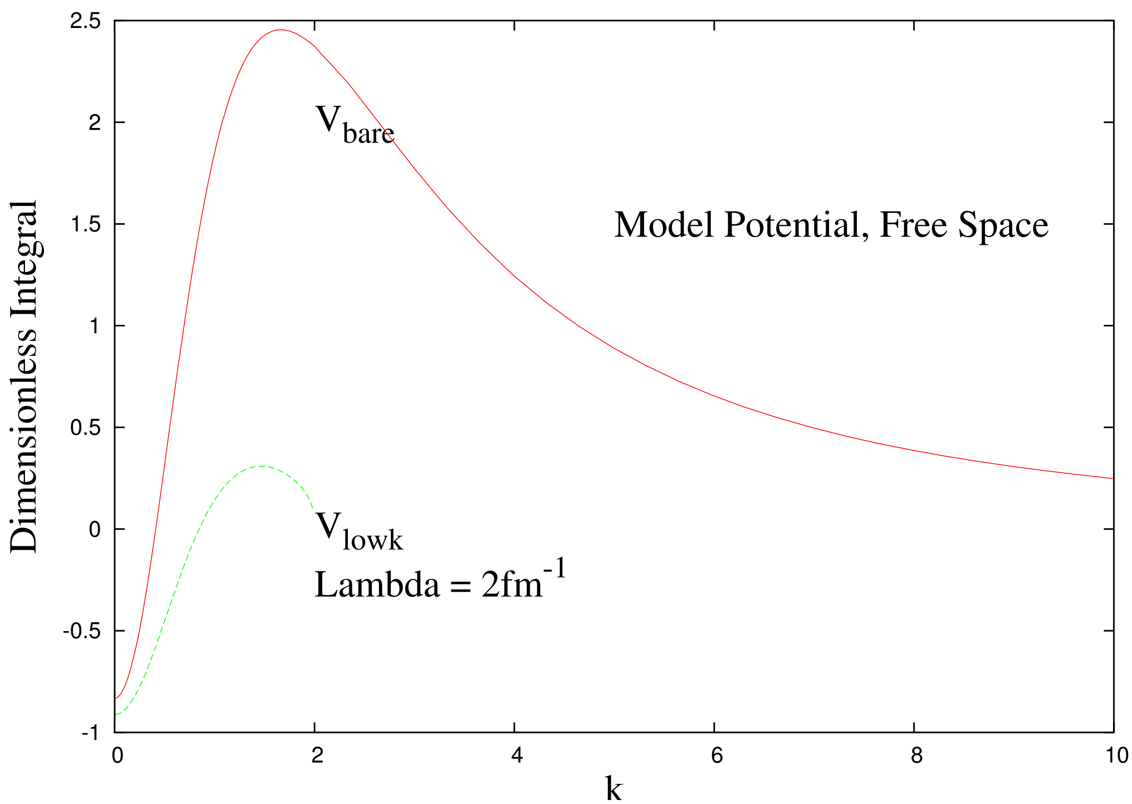


- Analytically for a sharp cut-off, relative error in $T(\Lambda, k; \Lambda^2)$ goes as $log(\epsilon) + \epsilon$.

- T matrix for a separable potential:

$$T(k',k;k^2) = \frac{L(k')R(k)}{1 + \frac{2}{\pi} P \int_0^{\Lambda} q^2\, dq \frac{L(q)R(q)}{k^2 - q^2}}$$

- Question: Is $T(k',k;k^2)$ perturbative? $\rightarrow$ use the integral in the denominator as a measure of perturbativeness.



- Calculate bulk property: $\frac{E_{gs}}{A}$ using the independent pair approximation

$$\frac{E_{gs}}{A} = \sum_{\vec{k_1}\vec{k_2}}^{k_F} \epsilon_{kin}(\vec{k_1},\vec{k_2}) + \sum_{\vec{k_1}\vec{k_2}}^{k_F} \Delta\epsilon_{\vec{k_1},\vec{k_2}}$$

- Switch variables to $\vec{k}$ and $\vec{P}$, where $\vec{P} = \vec{k_1} + \vec{k_2}$ and $\vec{k} = \frac{\vec{k_1} - \vec{k_2}}{2}$

$$\frac{E_{gs}}{A} = \sum_{\vec{P}\vec{k}}^{k_{max},P_{max}} \epsilon_{kin}(\vec{P},\vec{k}) + \sum_{\vec{P}\vec{k}}^{k_{max},P_{max}} \Delta\epsilon_{\vec{P},\vec{k}}$$

- Energy Shift $\Delta\epsilon_{\vec{P},\vec{k}}$ is related to the G matrix:

$$\Delta\epsilon_{\vec{P},\vec{k}} = \frac{G(\vec{k},\vec{k};k^2Pk_F)}{V}$$

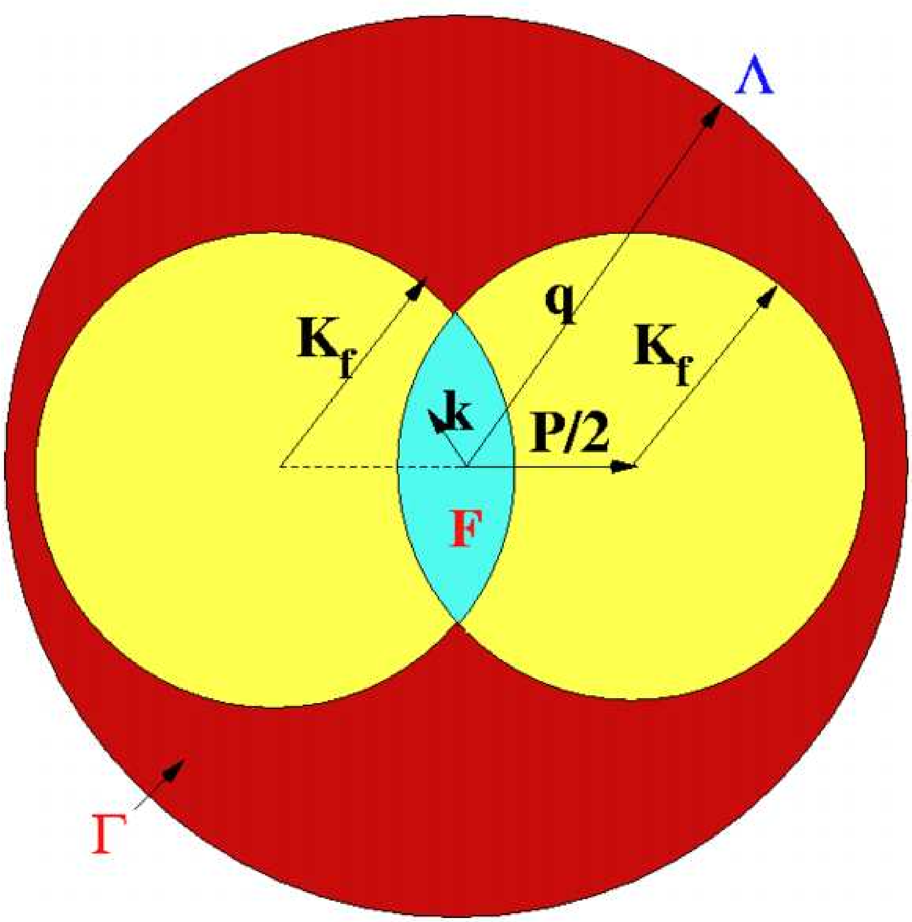
- For the l^{th} partial wave:

$$\Delta\epsilon^{l}_{P,k} = \frac{G_l(k,k;k^2Pk_F)}{V}$$

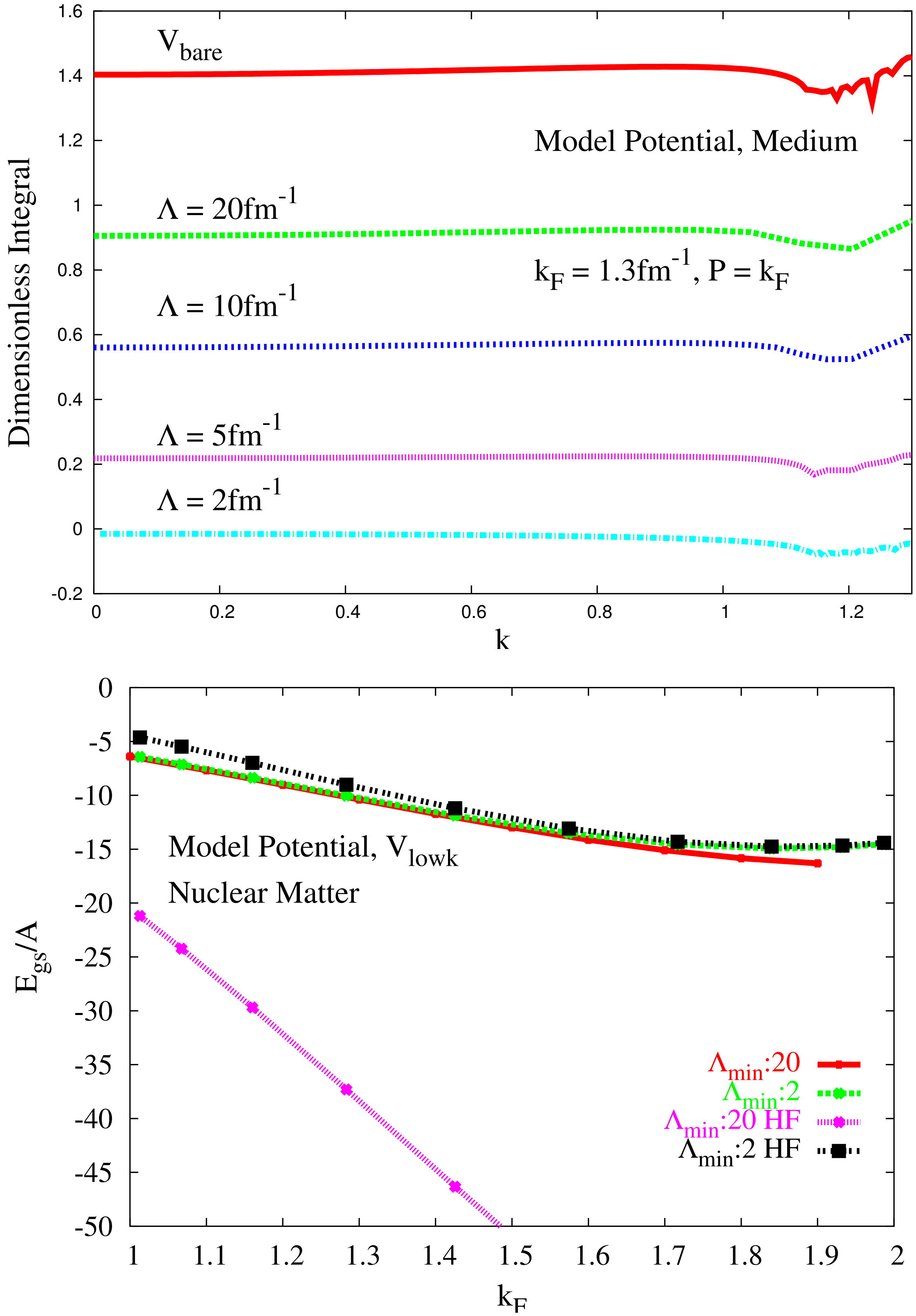
- For the l^{th} partial wave:

$$G_l(k,k;k^2Pk_F) = \frac{V^{\Lambda}(k,k)}{1+\frac{2}{\pi}\frac{m^*}{m}\int_{\Gamma}^{\Lambda}\frac{q^2V^{\Lambda}(q,q)}{q^2-k^2}}$$

- What is the region of integration?



RESULTS USING V^{Λ} setting $\frac{m^*}{m} = 1$



- SUMMARY
  - We have used R.G. as a tool for obtaining low-momentum potentials
  - Understanding the regulator: ϵ dependence
  - V^{Λ} for many-body calculation
  - Particle-Particle channel looks perturbative in medium

- FUTURE DIRECTIONS ..
  - Calculate $\frac{m^*}{m}$ self-consistently and do a better many-body calculation
  - understand the role of three-body forces