Covariant Effective Field Theory for a Dilute Fermi System

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- Effective action approach to EFT-based Kohn-Sham DFT
- Renormalization for short-range interactions
- Density functional theory (DFT) single-particle spectra

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Covariant DFT as Legendre Transformation

- To probe the system, add a source $V^\mu(x)$ coupled to current operator $\widehat{j}^\mu(x) \equiv \overline{\psi}(x)\gamma^\mu\psi(x)$ to the partition function:

$$\mathcal{Z}[V] = e^{-W[V]} \sim \mathrm{Tr}\, e^{-\beta(\widehat{H}+V\cdot\widehat{j})} \longrightarrow \int \mathcal{D}[\psi^\dagger]\mathcal{D}[\psi]\, e^{-\int[\mathcal{L}+V_\mu\,\overline{\psi}\gamma^\mu\psi]}$$

- The (time-dependent) current $j^\mu(x)$ in the presence of $V^\mu(x)$ is

$$j^\mu(x) = \big(\rho_V(x), \mathbf{j}_V(x)\big) \equiv \langle\overline{\psi}(x)\gamma^\mu\psi(x)\rangle_V = \frac{\delta W[V]}{\delta V_\mu(x)}$$

- Invert to find $V^\mu[j]$ and Legendre transform from V^μ to j^μ:

$$\Gamma[j] = -W[V] + \int V\cdot j \quad \text{with} \quad V^\mu(x) = \frac{\delta\Gamma[j]}{\delta j_\mu(x)} \longrightarrow \left.\frac{\delta\Gamma[j]}{\delta j_\mu(x)}\right|_{j_{\rm gs}(\mathbf{x})} = 0$$

$\Longrightarrow$ For static $j^\mu(\mathbf{x})$, $\Gamma[j] \propto$ the DFT energy functional $E[\rho_V]$!

What About the Scalar Density?

- Can add additional sources and Legendre transformations
- In nonrelativistic DFT, add to Lagrangian $+\eta(x)\,\nabla\psi^\dagger\nabla\psi$

$$\Gamma[\rho,\tau] = W[J,\eta] - \int J(x)\rho(x) - \int \eta(x)\tau(x)$$

$\Longrightarrow$ Skyrme HF energy functional $E[\rho,\tau,\mathbf{J}]$ of density *and* kinetic energy density (see A. Bhattacharyya talk DF06)

- In covariant DFT, add to Lagrangian $+\,S(x)\,\overline{\psi}\psi$

$$\Gamma[j^\mu,\rho_s] = W[V^\mu,S] - \int V(x)\cdot j(x) - \int S(x)\rho_s(x)$$

$\Longrightarrow$ RMF energy functional $E[\rho_v,\rho_s]$ [with $j^\mu = (\rho_v,\mathbf{0})$]

Kohn-Sham DFT Via Inversion [Fukuda et al.]

- Hierarchy (e.g., EFT expansion) $\Longrightarrow$ order-by-order matching

$$\begin{aligned} W[V] &= W_0[V] + W_1[V] + W_2[V] + \cdots \\ V[\rho_v] &= V_0[\rho_v] + V_1[\rho_v] + V_2[\rho_v] + \cdots \\ \Gamma[\rho_v] &= \Gamma_0[\rho_v] + \Gamma_1[\rho_v] + \Gamma_2[\rho_v] + \cdots \end{aligned}$$

- Zeroth order is a noninteracting system with potential $V_0(x)$

$$\Gamma_0[\rho_v] = W_0[V_0] - \int d^4x\, V_0(x)\rho_v(x) \quad \Longrightarrow \quad \rho_v(x) = \frac{\delta W_0[V_0]}{\delta V_0(x)}$$

$\Longrightarrow$ this is the Kohn-Sham system with the exact density!

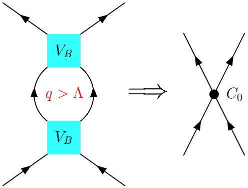
- Diagonalize $W_0[V_0]$ by introducing KS orbitals $\Longrightarrow$ Dirac equation
- Find V_0 for the ground state by completing self-consistency loop:

$$V_0 \to W_1 \to \Gamma_1 \to V_1 \to W_2 \to \Gamma_2 \to \cdots \Longrightarrow V_0(\mathbf{x}) = \frac{\delta \widetilde{E}_{\rm int}[\rho_v]}{\delta \rho_v(\mathbf{x})} \propto \sum_i \frac{\delta \Gamma_i[\rho_v]}{\delta \rho_v(\mathbf{x})}$$

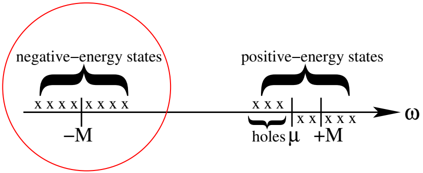
Power Counting Lost / Power Counting Regained

- Gasser, Sainio, and Svarc $\Longrightarrow$ ChPT for πN with relativistic N's
  - loop and momentum expansions don't agree
    $\Longrightarrow$ systematic power counting lost
  - heavy-baryon EFT restores power counting by $1/M$ expansion
- Hua-Bin Tang (1996) [and with Paul Ellis]:

  "... EFT's permit useful low-energy expansions only if we absorb all of the hard-momentum effects into the parameters of the Lagrangian."



and

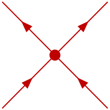


- Becher/Leutwyler IR $\Longrightarrow$ Schindler-Gegelia-Scherer version

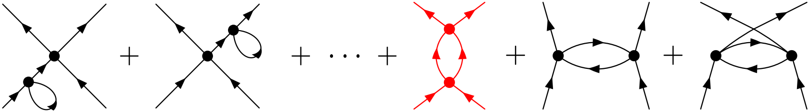
Consequences for Free-Space Natural Fermions

- Tadpoles, $N\overline{N}$ loops in free space vanish!
- Leading order (LO) has scalar, vector, etc. vertices

$$\mathcal{L}_{\rm eft} = \cdots - \frac{C_s}{2}(\overline{\psi}\psi)(\overline{\psi}\psi) - \frac{C_v}{2}(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi) + \cdots \Longrightarrow$$



- At NLO, only particle-particle loop survives IR



- Only forward-going nucleons contribute

  $\Longrightarrow$ same scattering amplitude as nonrel. DR/MS for small k

Laboratory for Covariant DFT

- Study covariant vs. nonrel. DFT/EFT in controlled expansion
  - Confine system in a Woods-Saxon "trap" with $V_{\rm ext}$ and $S_{\rm ext}$
  - Choose $V_{\rm ext}, S_{\rm ext}$ so no spin-orbit from external potentials
- First: Consider LO covariant effective Lagrangian

$$\mathcal{L}_{\rm eft} = \overline{\psi}[i\partial_\mu\gamma^\mu - M]\psi - \frac{C_s}{2}(\overline{\psi}\psi)(\overline{\psi}\psi) - \frac{C_v}{2}(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi)$$

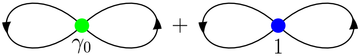
- Compare observables from two DFT functionals
  - Sources coupled to vector $V^\mu\overline{\psi}\gamma_\mu\psi$ and scalar $S\overline{\psi}\psi$ densities
    $\Longrightarrow$ two Kohn-Sham potentials, scalar and vector

$$V_0^\mu(\mathbf{x}) = \frac{\delta E_{\rm int}[j^\mu, \rho_s]}{\delta j_\mu(\mathbf{x})} \quad \text{and} \quad S_0(\mathbf{x}) = \frac{\delta E_{\rm int}[j^\mu, \rho_s]}{\delta \rho_s(\mathbf{x})}$$

  - Source coupled to vector $\widetilde{V}^\mu\overline{\psi}\gamma_\mu\psi$ densities only
    $\Longrightarrow$ one (vector) Kohn-Sham potential (plus external fields)

$$\widetilde{V}_0^\mu(\mathbf{x}) = \frac{\delta \widetilde{E}_{\rm int}[j^\mu]}{\delta j_\mu(\mathbf{x})}$$

- Consider bowtie diagrams with four-vector and scalar vertices:

$\Gamma_1 = W_1 =$ 

- Case ρ_v, ρ_s: Quadratic part of Lagrangian in W_0 diagonalized

$$\int d^4x\, \overline{\psi}\Big[i\partial^\mu\gamma_\mu - M + S_0(\mathbf{x}) - \gamma_0 V_0^0(\mathbf{x})\Big]\psi$$

  - Kohn-Sham equation $\Longrightarrow$ defines $M^*(\mathbf{x}) \equiv M - S_0(\mathbf{x})$

- Case ρ_v: Quadratic part of Lagrangian in $\widetilde{W}_0$ diagonalized

$$\int d^4x\, \overline{\psi}\Big[i\partial^\mu\gamma_\mu - M - \gamma_0 \widetilde{V}_0^0(\mathbf{x})\Big]\psi$$

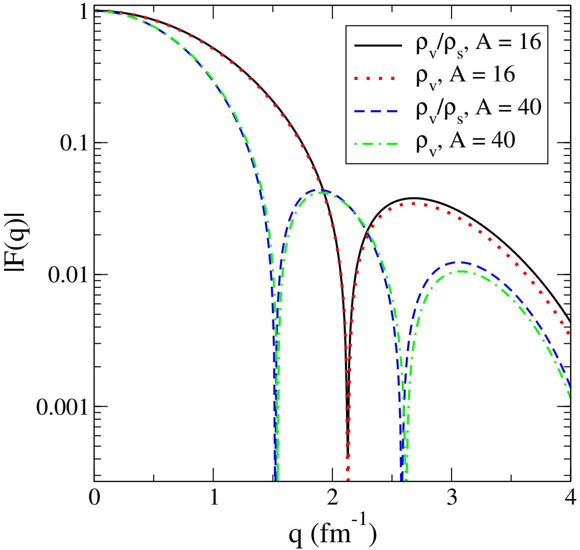
  - Scalar diagram evaluated approximately (k_{F}/M expansion + LDA)

- How do observables compare?
  - Binding energies, densities (Kohn-Sham observables)
  - What about the single-particle spectra?

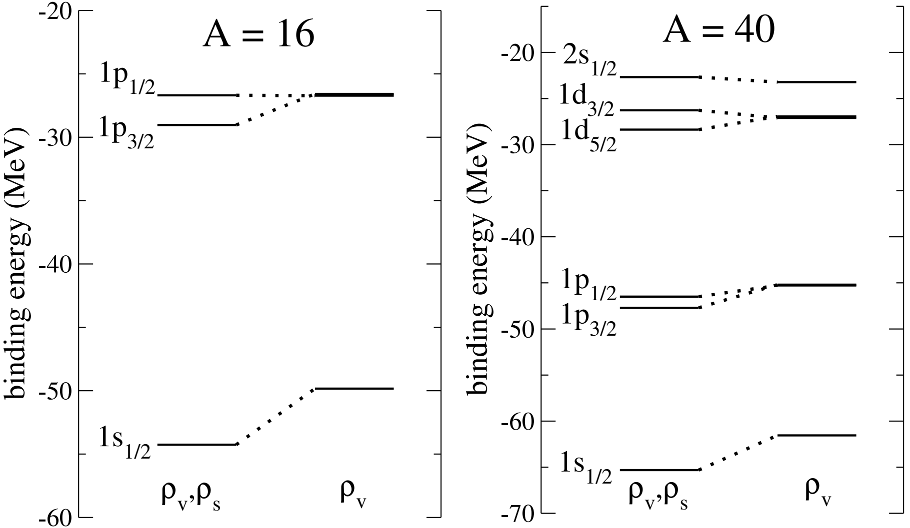
Kohn-Sham LDA ρ_V vs. ρ_V, ρ_S: Comparison

$A = 16$	BE/A	$\sqrt{\langle r^2 \rangle}$
ρ_V	27.5	1.97
ρ_V, ρ_S	27.1	1.99

$A = 40$	BE/A	$\sqrt{\langle r^2 \rangle}$
ρ_V	29.4	2.57
ρ_V, ρ_S	27.8	2.56



Energy Spectra ρ_V, ρ_S vs. ρ_V



Summary and Future

- Covariant fermions with short-range interactions in trap
  - Laboratory for studying covariant EFT, DFT issues
  - Infrared regularization (IR) restores power counting
  - IR greatly simplifies free space and medium
  - Kohn-Sham Single-particle properties depend on sources
    $\Longrightarrow$ can also relate to exact Green's function
- Issues under consideration:
  - Matching to free-space scattering
  - Comparison to heavy-baryon expansion
  - Renormalization at NLO
  - Covariant two-body $\Longrightarrow$ non-relativistic three-body forces
  - Time-dependent DFT (cf. RPA)
  - Including pairing
  - $\ldots$

Summary Comments on Vacuum Physics

- Unlike QED DFT, “no sea” for nuclear structure is a misnomer
  - include “vacuum physics” in coefficients through renormalization
- Renormalization versus Renormalizability
  - Renormalization is required to account for short-distance behavior but can be implicit
  - Renormalizability at the hadronic level corresponds to making a model for the short-distance behavior
    - not a good model phenomenologically
  - Fixing short-distance behavior is not the same thing as throwing away negative-energy states
- For a long time, we looked for *unique* “relativistic effects”; these were largely misguided efforts

Why Use EFT for Energy Functionals

- Similar to conventional "phenomenological" approaches
  - but with a rigorous foundation (DFT from effective action)
  - extendable and can be connected to chiral EFT for NN/NNN
- Eliminating model dependences (cf. "minimal" models)
  - framework for building a "complete" functional
  - more efficient renormalization (e.g., pairing)
- New insight into analytic structure of functional
  - e.g., logs in low-density expansion in $k_{\mathrm{F}} a_s$ from RG
- Power counting: what to sum at each order in an expansion
  - naturalness $\Longrightarrow$ estimates of truncation errors
  - evidence from Skyrme and RMF functionals for hierarchy
  - for covariant EFT, requires special renormalization

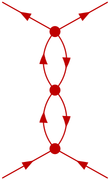
UV Divergences in Nonrelativistic and Relativistic Effective Actions

- *All* low-energy effective theories have incorrect UV behavior
- Sensitivity to short-distance physics signalled by divergences
  but finiteness (e.g., with cutoff) doesn't mean not sensitive!
  $\Longrightarrow$ must absorb (and correct) sensitivity via renormalization
- Instances of UV divergences

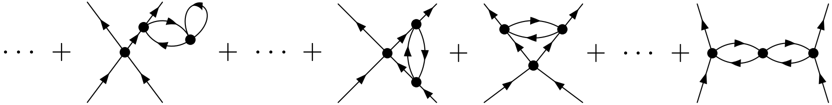
nonrelativistic	covariant
scattering	scattering
pairing	pairing
	anti-nucleons

Consequences for Free-Space Natural Fermions

- At NNLO, only particle-particle loop diagram survives IR



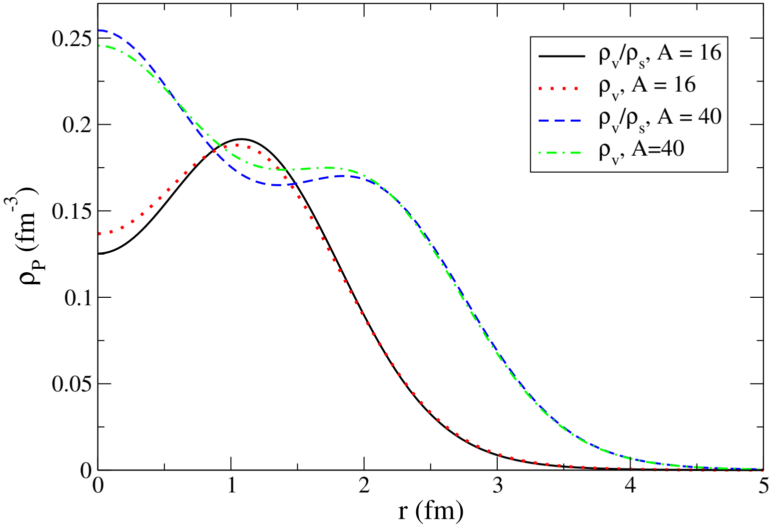
- All other diagrams are zero in IR



Moving Dirac Sea Physics into Coefficients

- Absorb the “hard” part of a diagram into parameters,
  $\Longrightarrow$ the remaining “soft” part satisfies chiral power counting
  - original πN prescription by H.B. Tang (expand, integrate term-by-term, and resum propagators)
  - systematized for πN by Becher and Leutwyler: “infrared regularization” or IR
  - not unique; e.g., Fuchs et al. additional finite subtractions in DR
- Extension of IR to multiple heavy particles [Lehmann and Prézeau]
  - convenient reformulation by Schindler, Gegelia, Scherer
  - tadpoles, $N\overline{N}$ loops in free space vanish!
  - particle-particle loop reduces to nonrelativistic DR/MS result

Kohn-Sham LDA ρ_V vs. ρ_V, ρ_S



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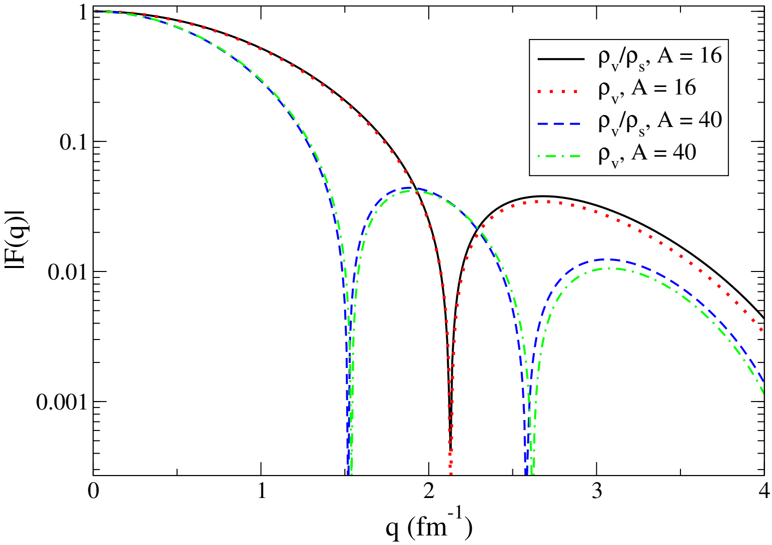

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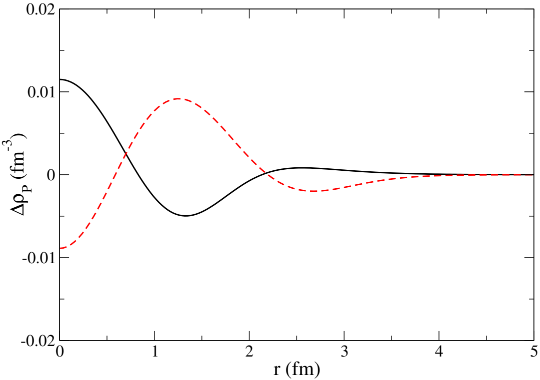
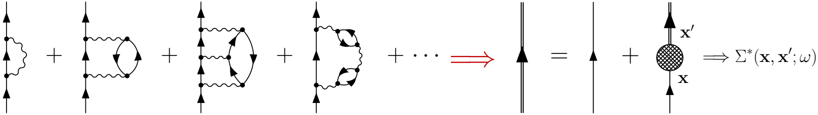


How is the Exact G Related to $G_{\rm ks}$?

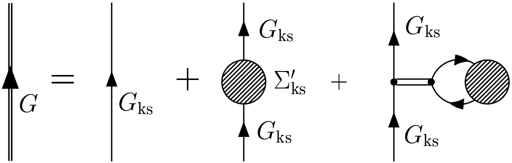


- Add a non-local source $\xi(x',x)$ coupled to $\overline{\psi}(x')\psi(x)$:

$$Z[V,\xi] = e^{iW[V,\xi]} = \int D\psi D\psi^\dagger \; e^{i\int d^4x\;[\mathcal{L} + V^\mu(x)\overline{\psi}(x)\gamma_\mu\psi(x) + \int d^4x'\;\xi(x',x)\overline{\psi}(x')\psi(x)]}$$

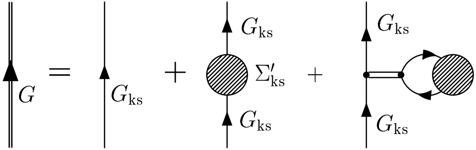
- With $\Gamma[\rho_v,\xi] = \Gamma_0[\rho_v,\xi] + \Gamma_{\rm int}[\rho_v,\xi]$ (Dirac indices suppressed),

$$G(x,x') = \left.\frac{\delta W}{\delta \xi}\right|_J = \left.\frac{\delta \Gamma}{\delta \xi}\right|_{\rho_v} = G_{\rm ks}(x,x') + G_{\rm ks}\left[\frac{\delta \Gamma_{\rm int}}{\delta G_{\rm ks}} - \frac{\delta \Gamma_{\rm int}}{\delta \rho_v}\right] G_{\rm ks}$$

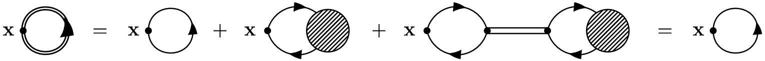


How Do G and G_{ks} Yield the Same Density?

- Claim: $\rho_{\mathrm{ks}}(\mathbf{x}) = -i\nu G^{0}_{\mathrm{KS}}(x, x^{+})$ equals $\rho(\mathbf{x}) = -i\nu G(x, x^{+})$
- Start with



- Simple diagrammatic demonstration:



- Densities agree by construction!
- But other quantities may differ; spectral functions?