# Three-Body Interactions in Many-Body Effective Field Theory

*RJF, The Ohio State University*

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  - ∗ general principles for low-energy effective theories
  - ∗ local Lagrangian EFT for many-fermion systems
  - ∗ pedagogical example: fermions with short-range interactions only

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  - $*$ local Lagrangian EFT for many-fermion systems
  - $*$ pedagogical example: fermions with short-range interactions only
- Inevitability of three-body interactions
  - $*$ renormalization group $\Longrightarrow$ analytic structure of energy density
- Model dependence in many-body observables (“off-shell ambiguities”)
  - $*$ occupation numbers: field redefinitions and three-body interactions

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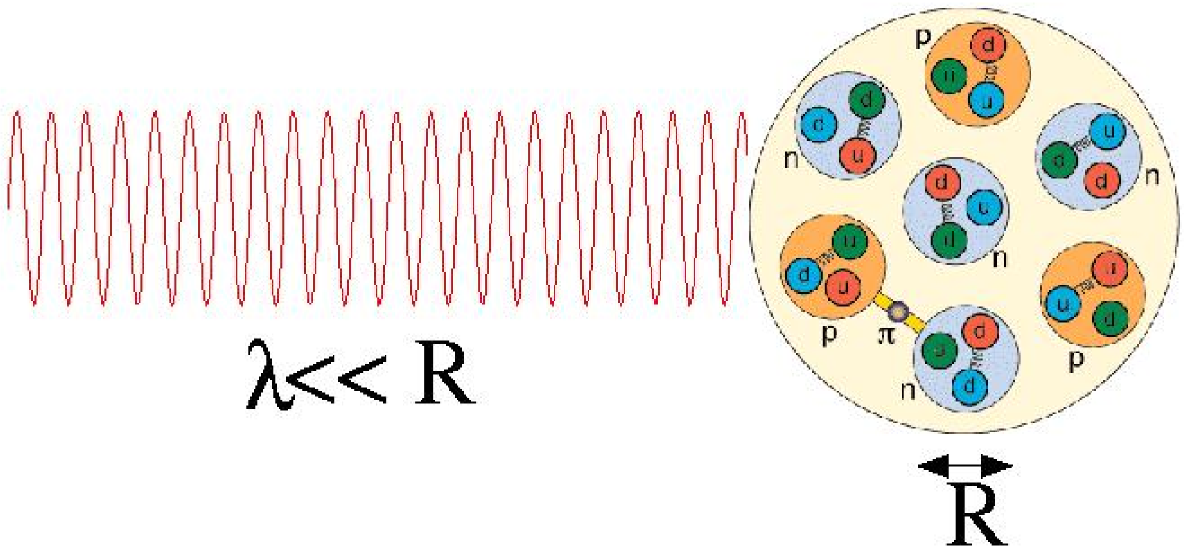
  - ∗ occupation numbers: field redefinitions and three-body interactions
- Current trends: Work in progress

# General Principle of Effective Low-Energy Theories

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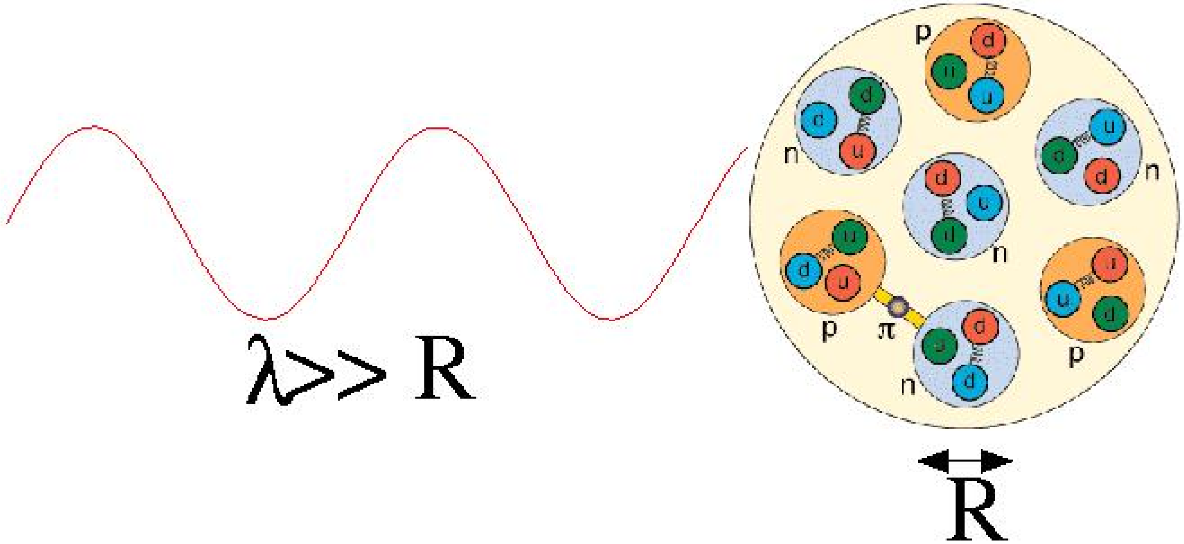


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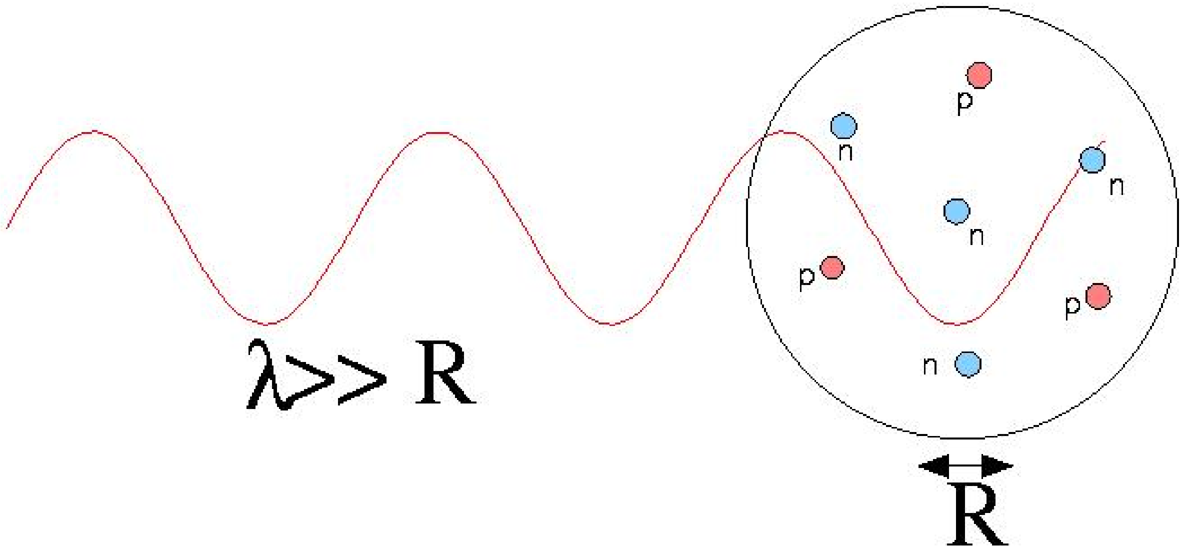


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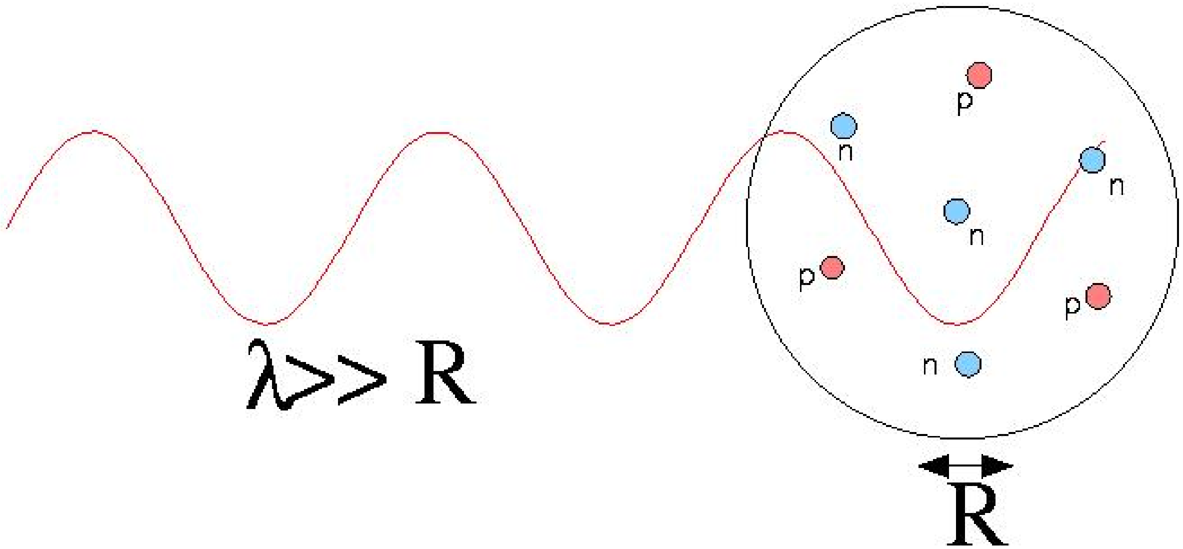


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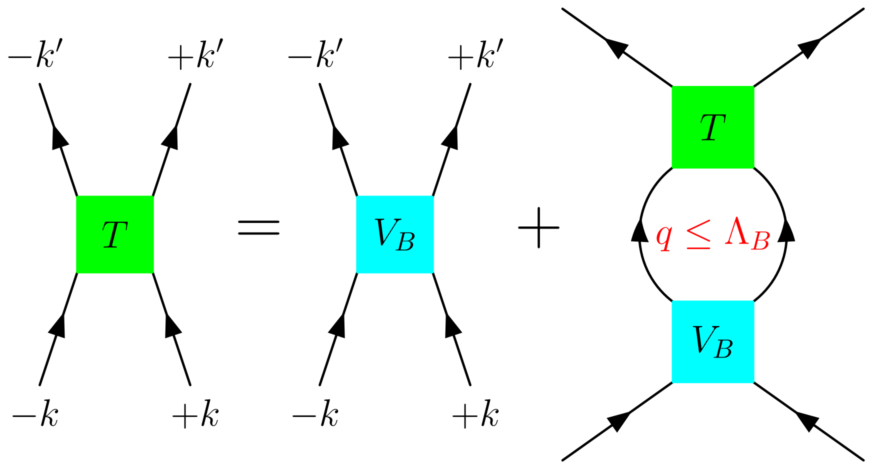
  - ∗ use low-energy dof's for low-energy processes (easier, more efficient)
  - ∗ short-distance structure can be replaced by something simpler (and wrong at short distances!) without distorting low-energy observables
- Many ways to replace structure; most physical is using cutoff $\Lambda$

# Low-Momentum NN Potential: Bogner, Kuo, Schwenk, et al.




- NN scattering in the COM frame
- $\chi^2/$dof $\approx 1$ “bare” potential $V_B$
- Probes intermediate states to

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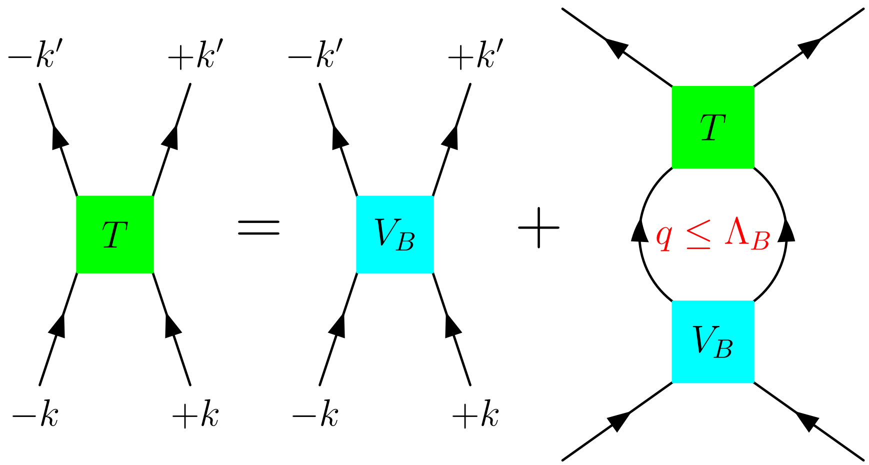


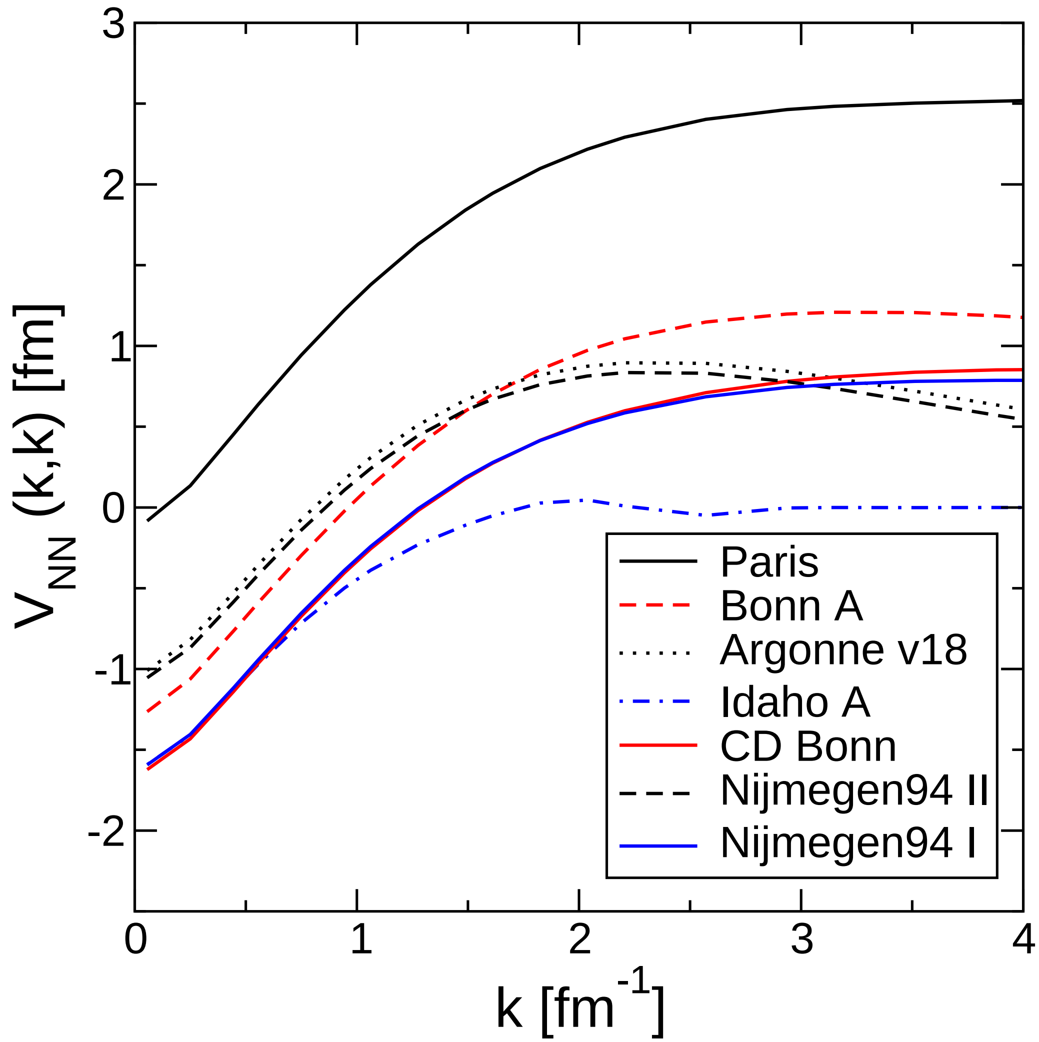


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- Model dependent: $q \geq 3\,\text{fm}^{-1}$

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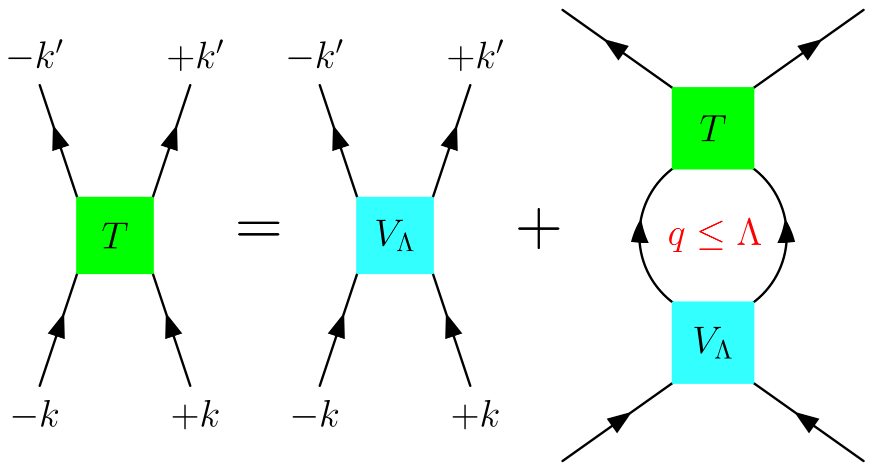


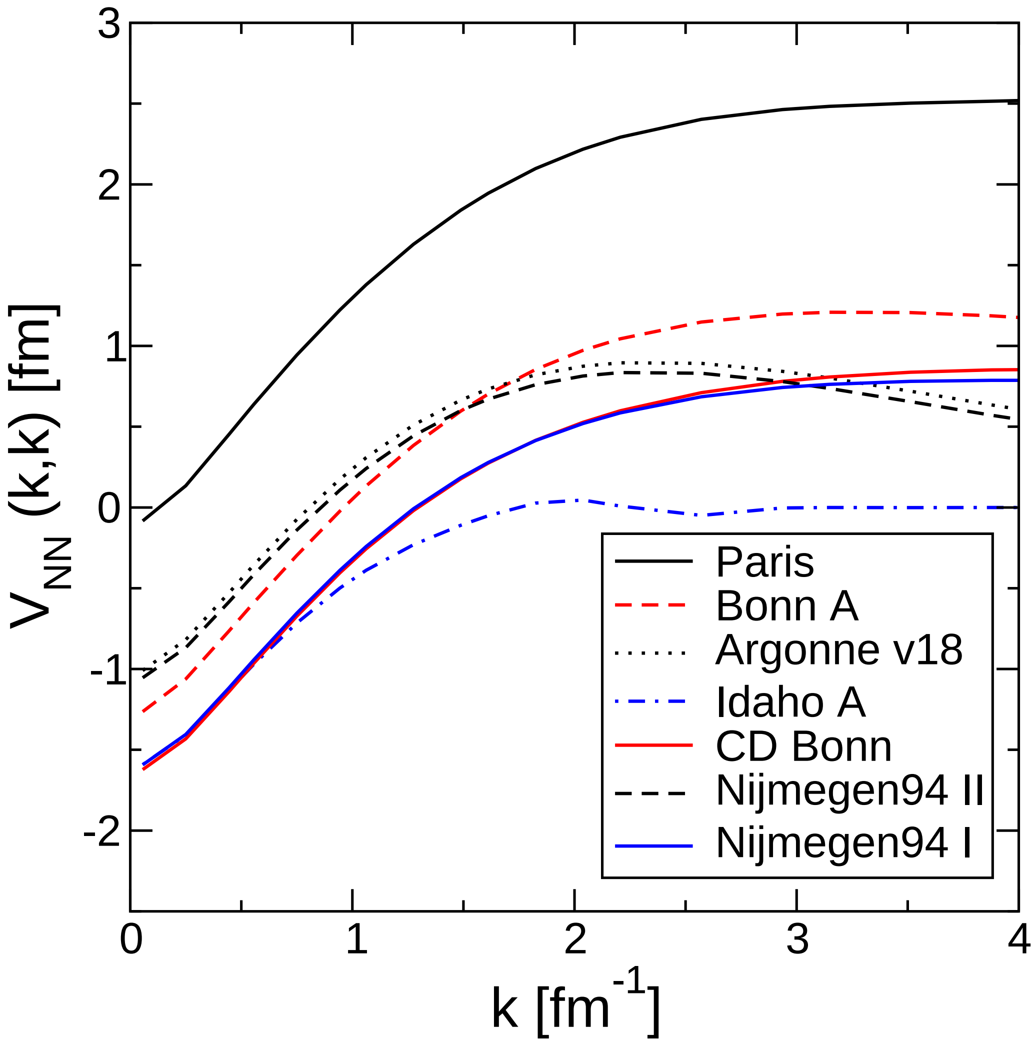

- Intermediate states only up to

  $q \leq \Lambda = 2\,\text{fm}^{-1}$

  $\Longrightarrow E_{\text{lab}} \doteq 350\,\text{MeV}$

- Require same phase shifts

  $\Longrightarrow V_B \rightarrow V_\Lambda$



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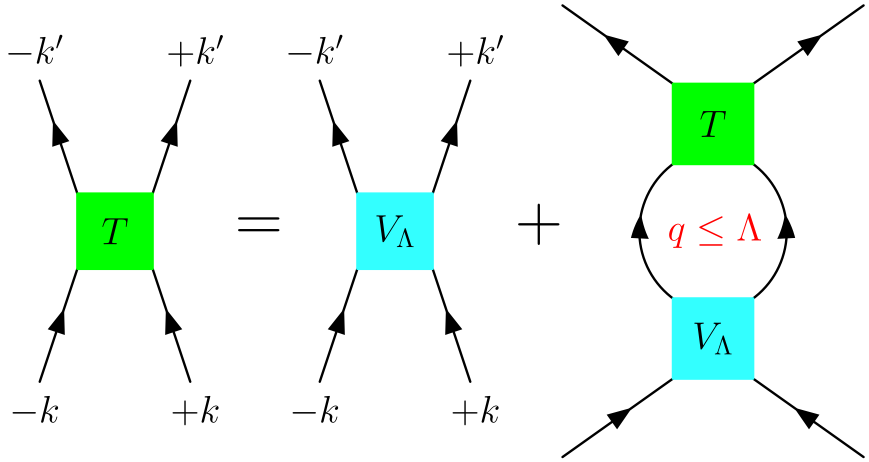


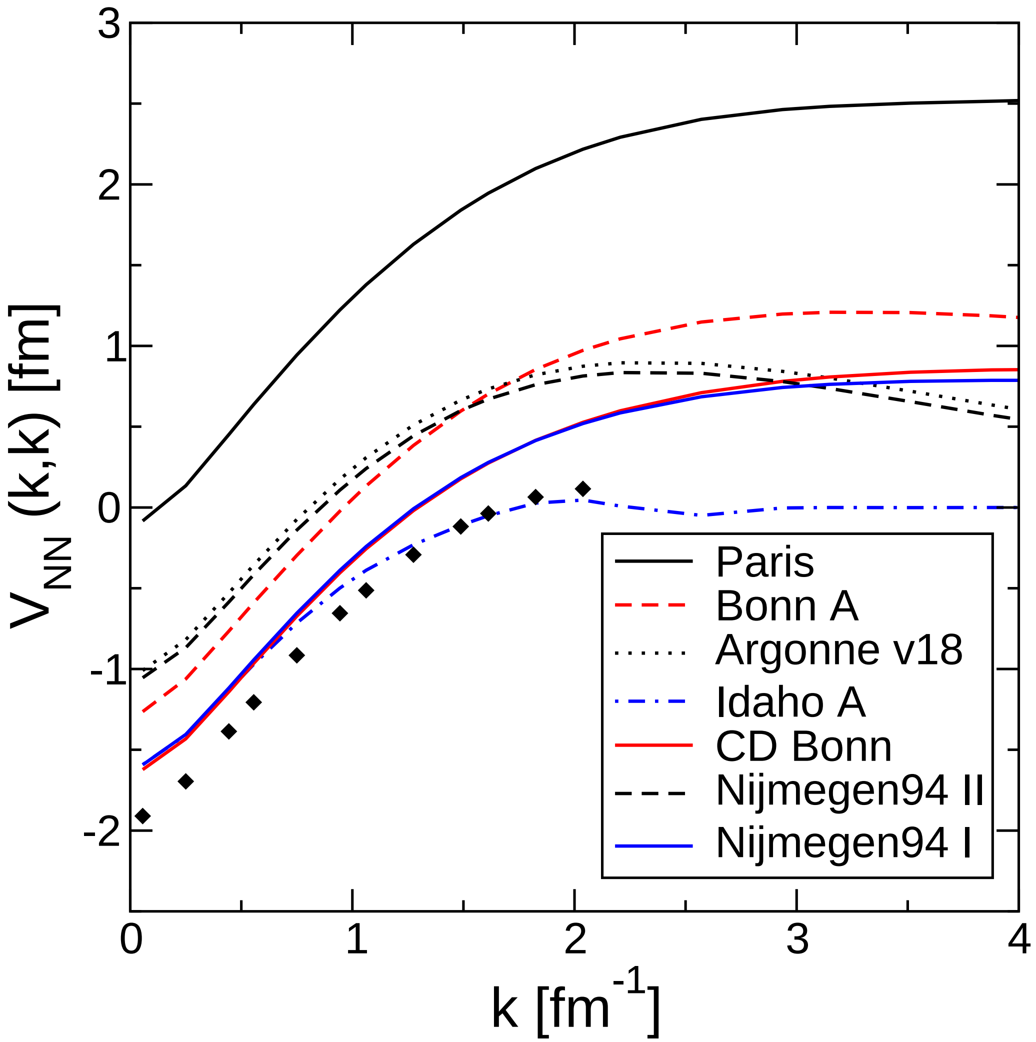


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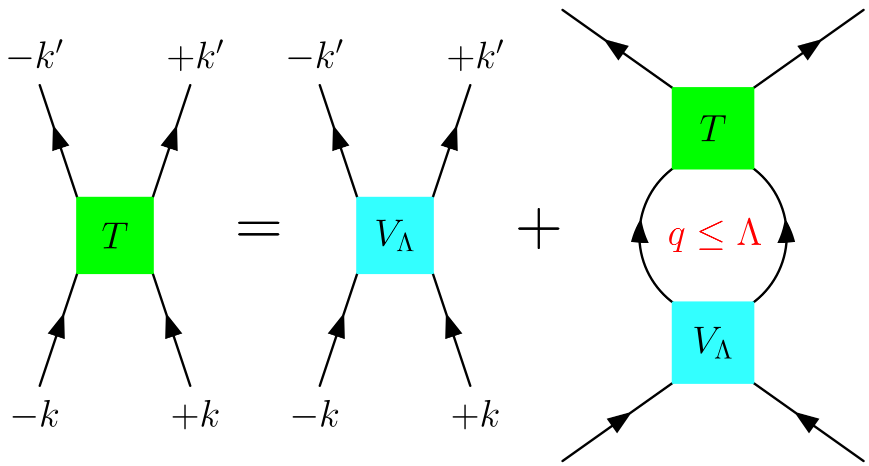


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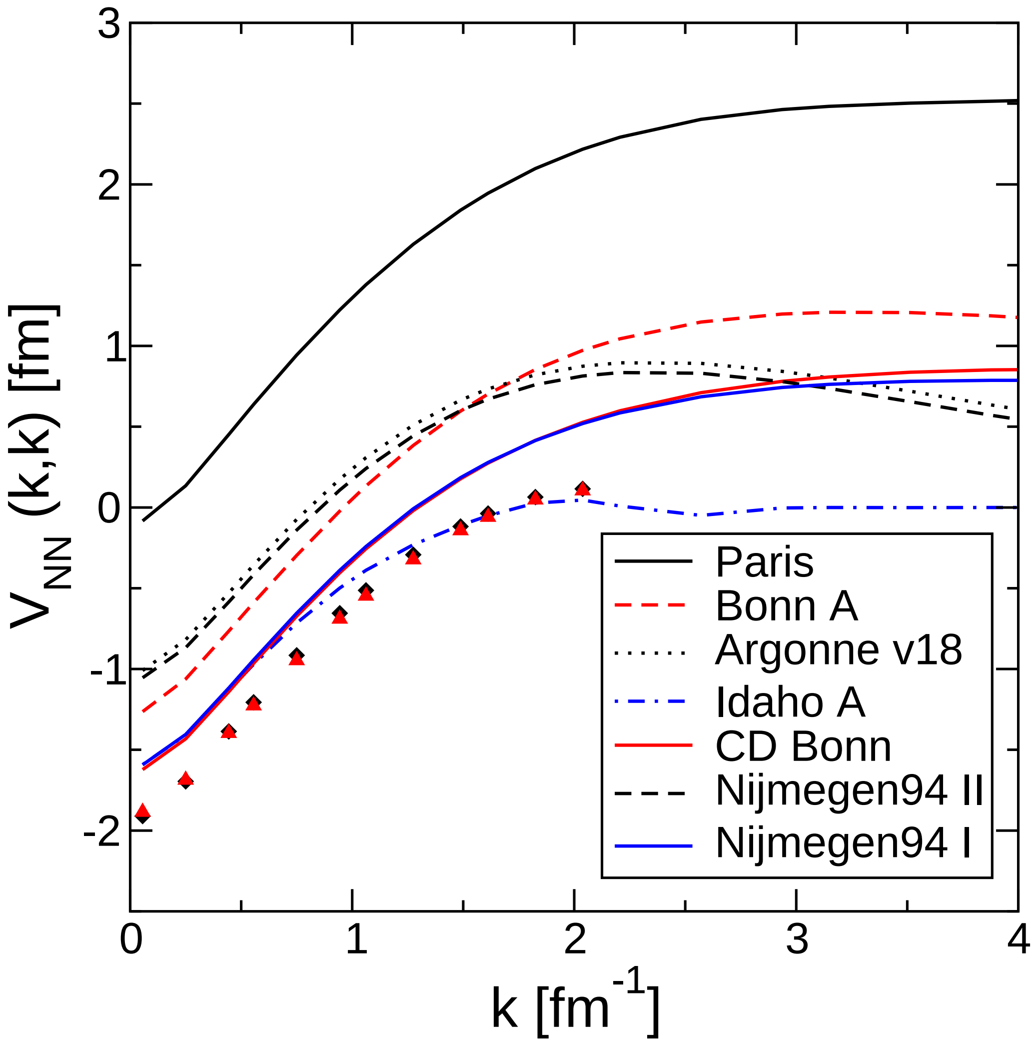

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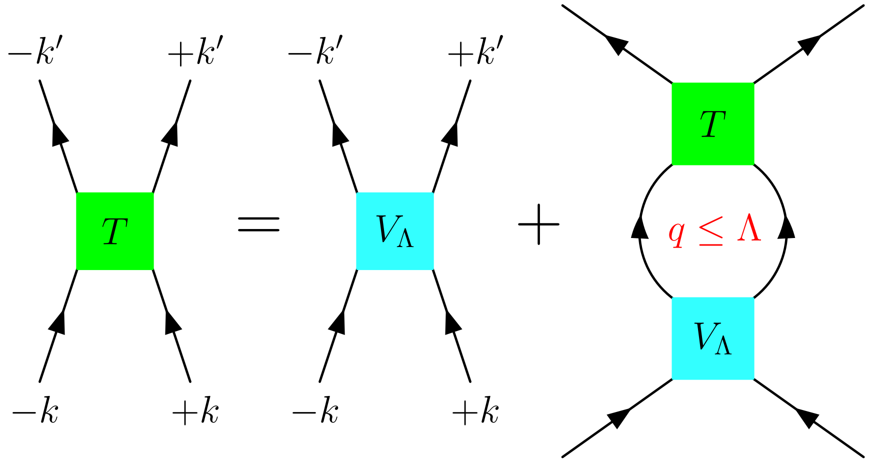


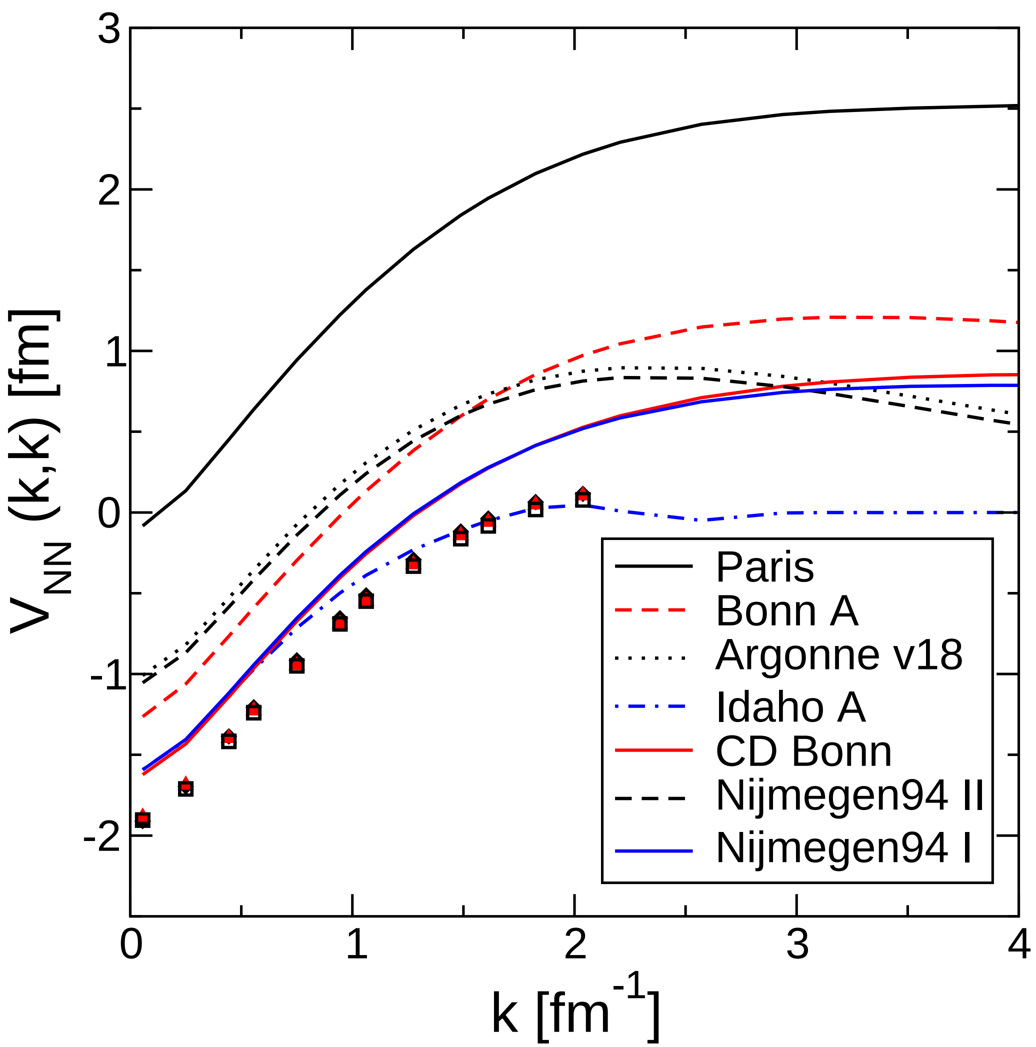


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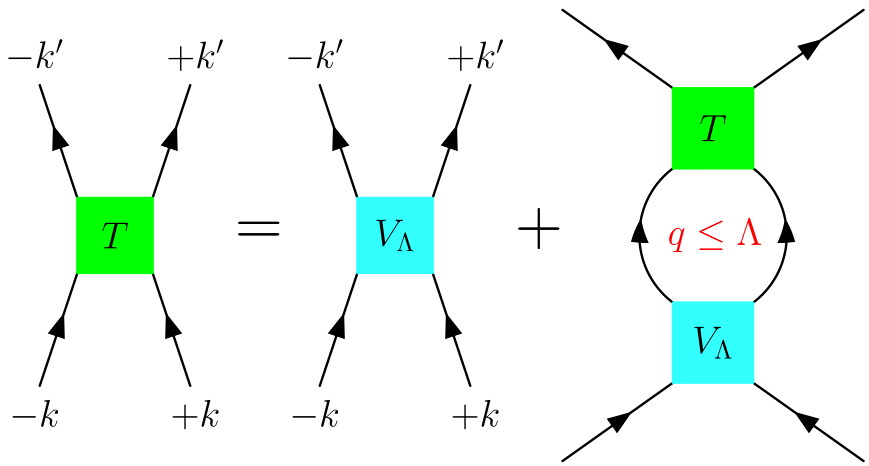


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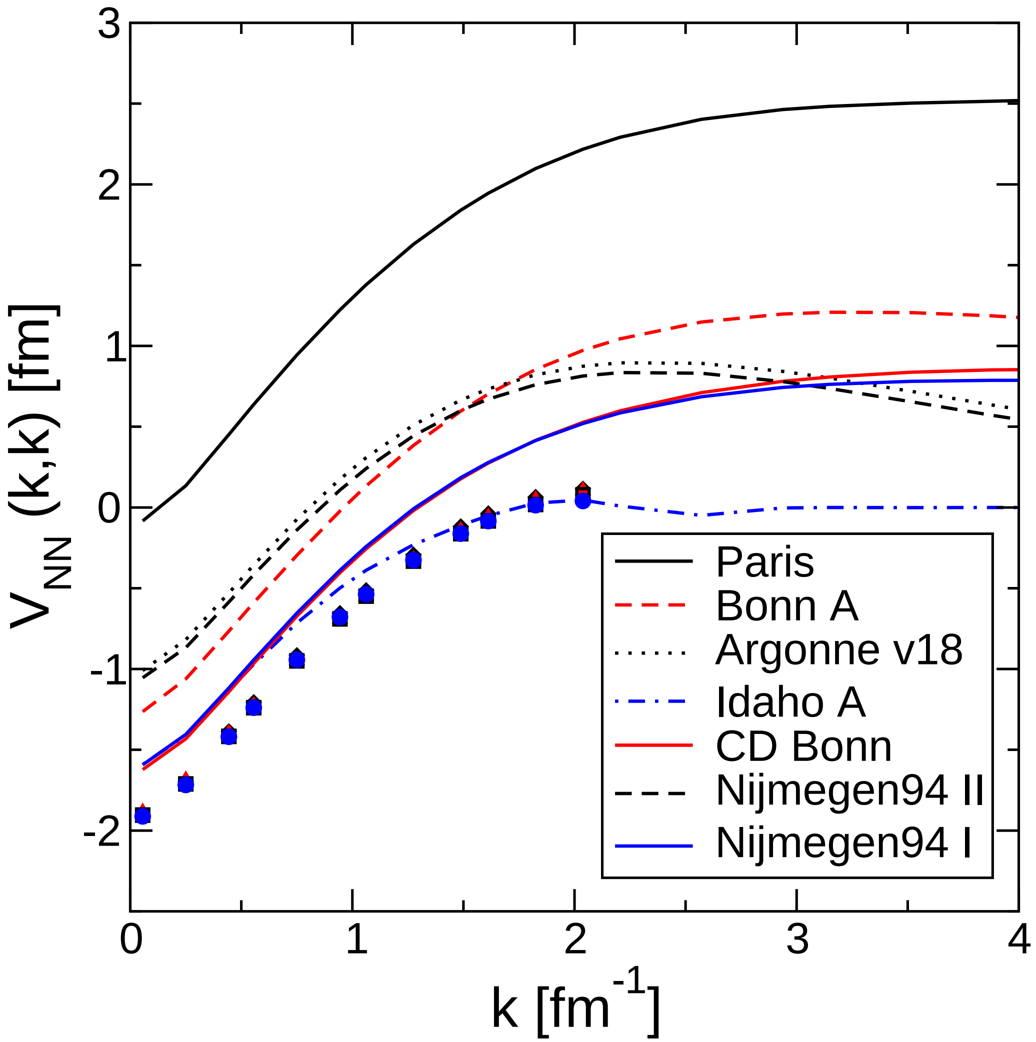

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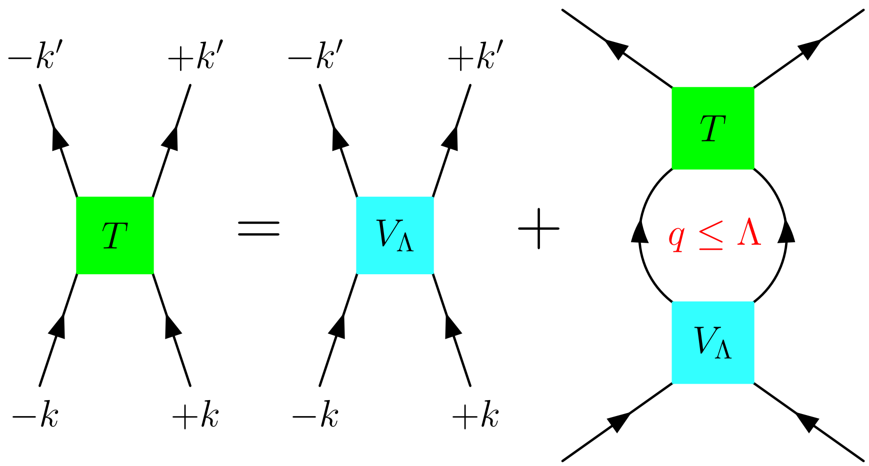


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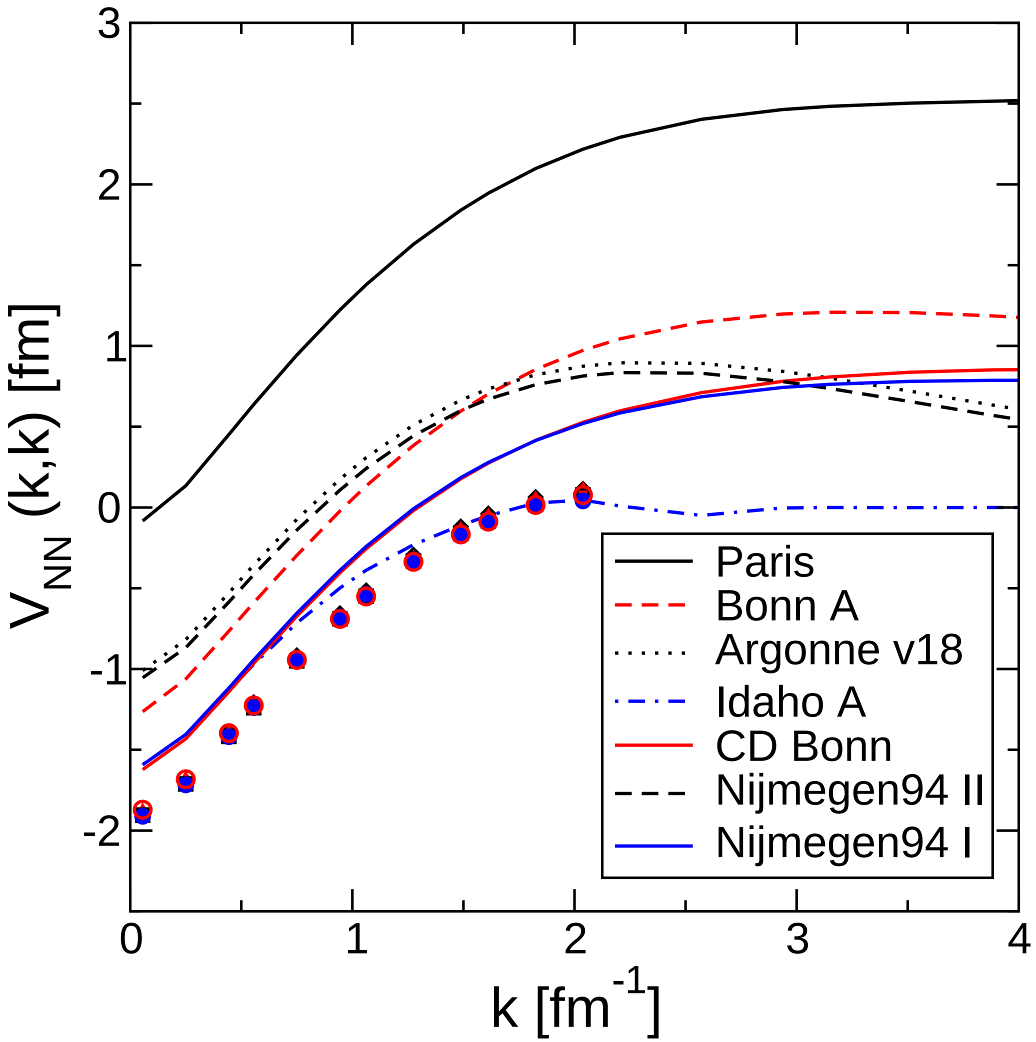

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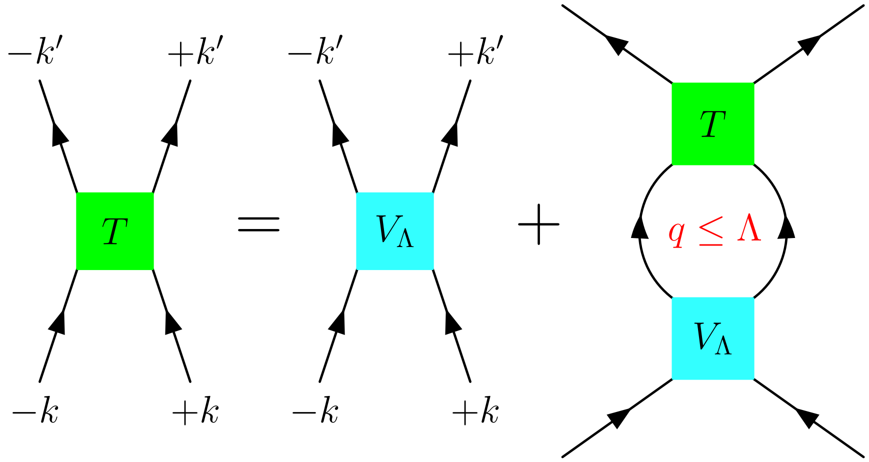


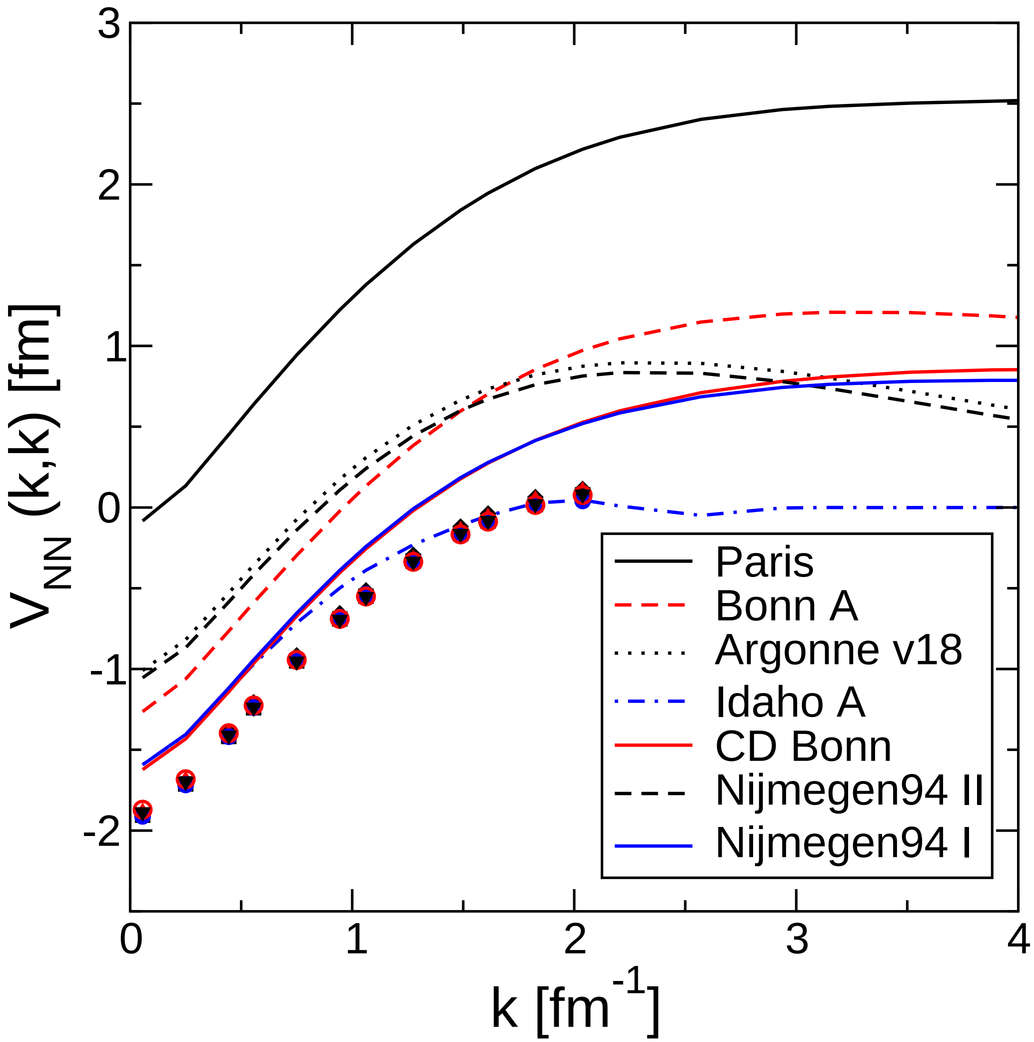


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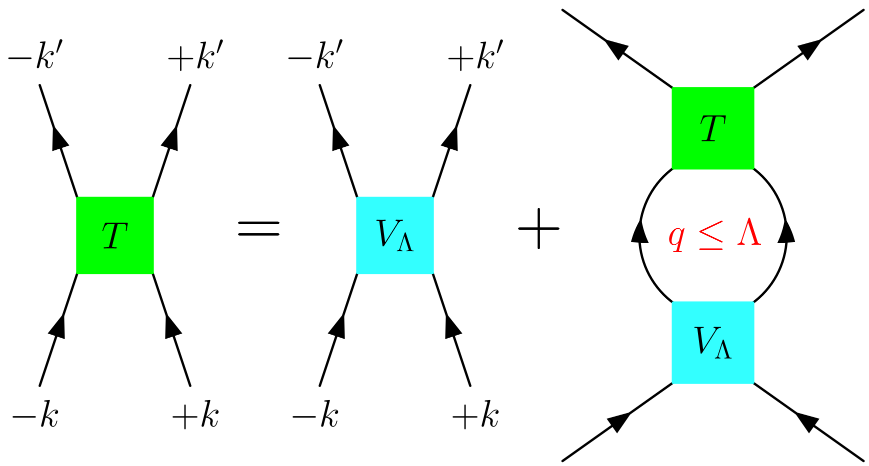


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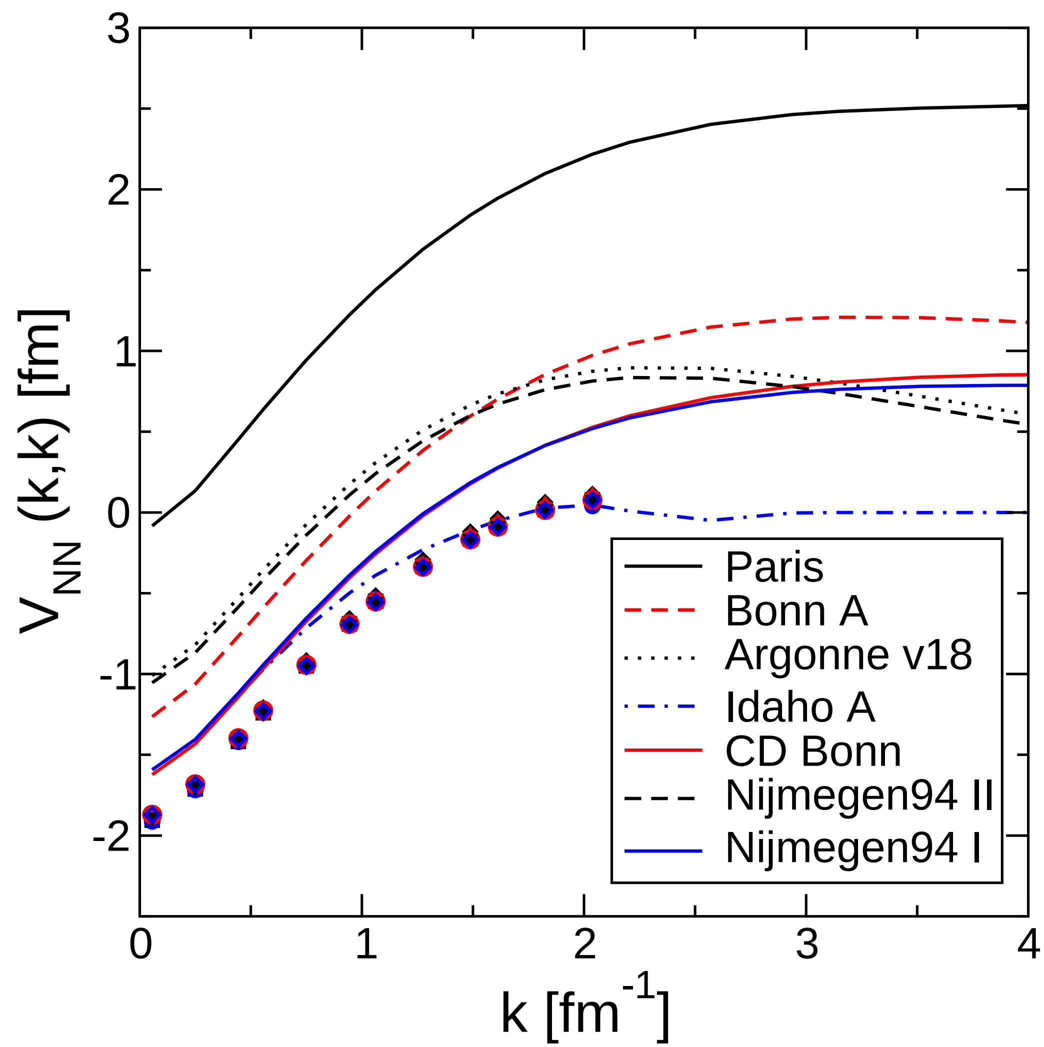
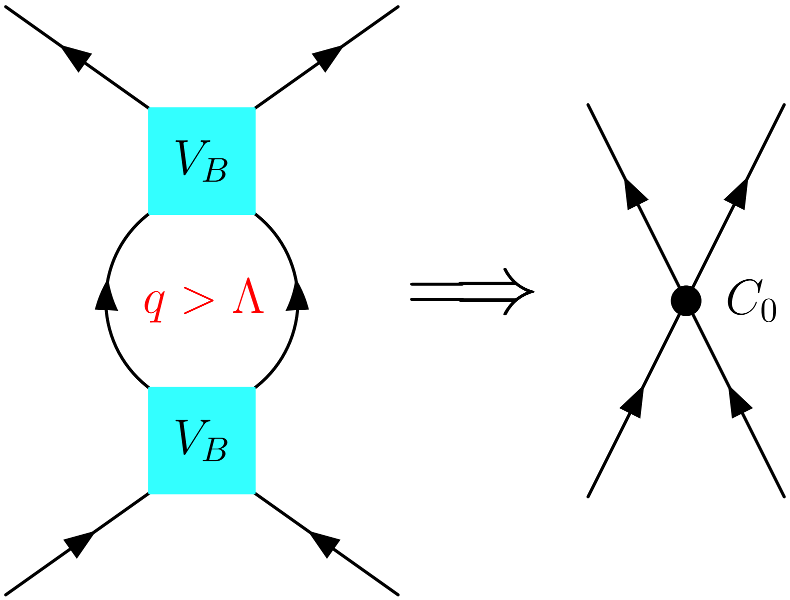
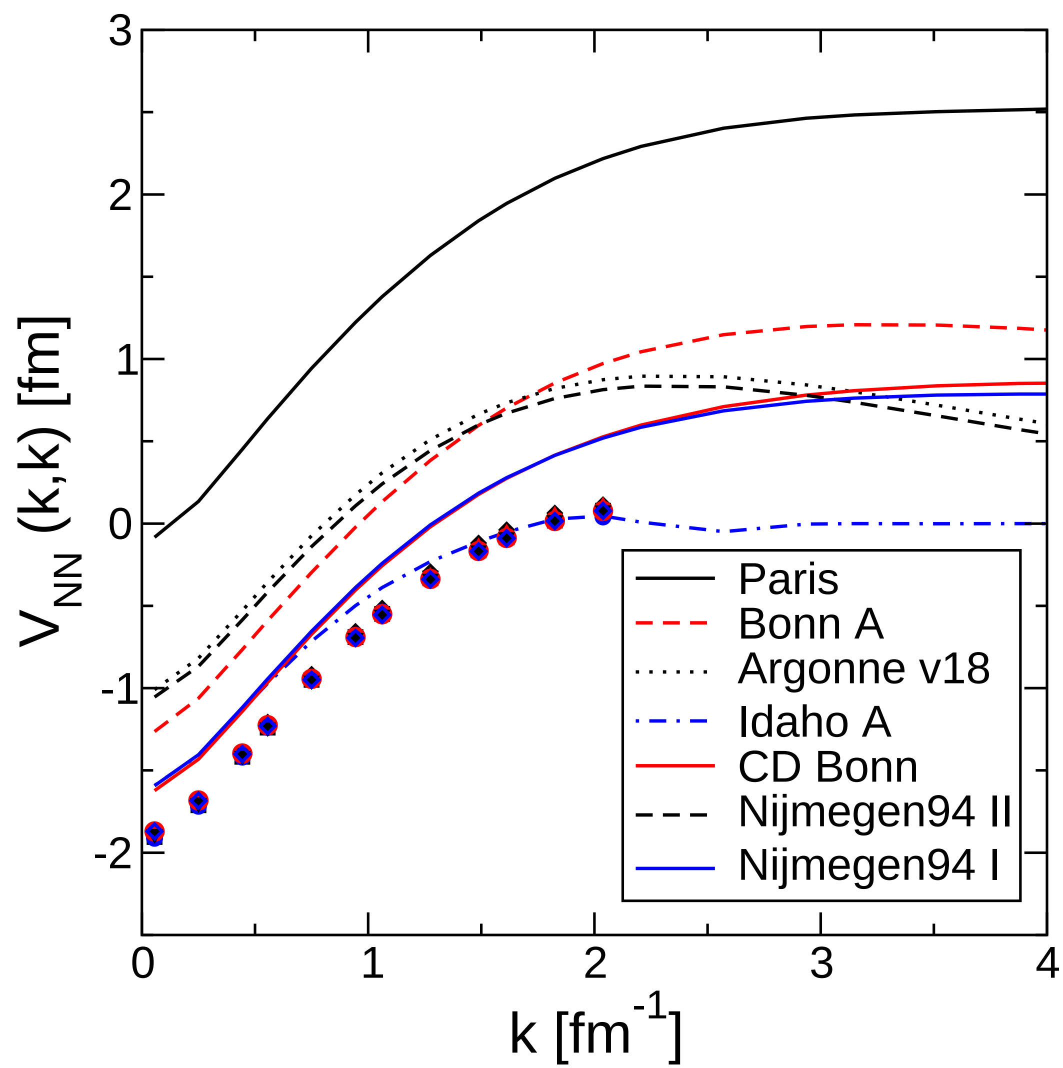


# Renormalization: Absorbing the Short-Distance Physics




- $q > \Lambda$ intermediate states
  $\Longrightarrow$ replace with contact term:
  $$C_0 \delta^3(\mathbf{x} - \mathbf{x}')$$
- $\mathcal{L}_{\text{eft}} = \cdots + \frac{1}{2}(\psi^\dagger \psi)^2 + \cdots$
- $2 \to 2$ short-range only
  $\Longrightarrow$ *power* divergences

## Lessons from $V_\Lambda \Longrightarrow$ Effective Field Theory

- Low-energy data insensitive to *details* of short-distance physics

  $\Longrightarrow$ replace by something simpler without distorting low-energy physics

  $*$ EFT is local Lagrangian, model-independent approach to this program

  - symmetries incorporated; consistent currents
  - separation of scales $\Longrightarrow$ expansion parameters
  - power counting $\Longrightarrow$ error estimates $\Longrightarrow$ systematic

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- Short-distance physics absorbed into local terms $\Longrightarrow$ Renormalization!

  $\ast$ in an EFT, shift between loops and low-energy constants (LEC's)

  $\ast$ long-range explicit (e.g., pion) + short-distance interactions

## Effective Field Theory Ingredients: Dilute Fermi System

- See “Crossing the Border” [nucl-th/0008064]

1. Use the most general $\mathcal{L}$ with low-energy dof’s consistent with global and local symmetries of underlying theory
   - $\ast$
2. Declaration of regularization and renormalization scheme
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3. Well-defined power counting $\Longrightarrow$ small expansion parameters
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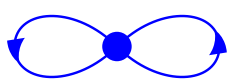
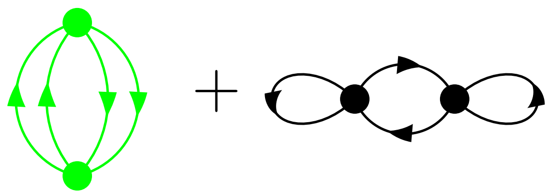
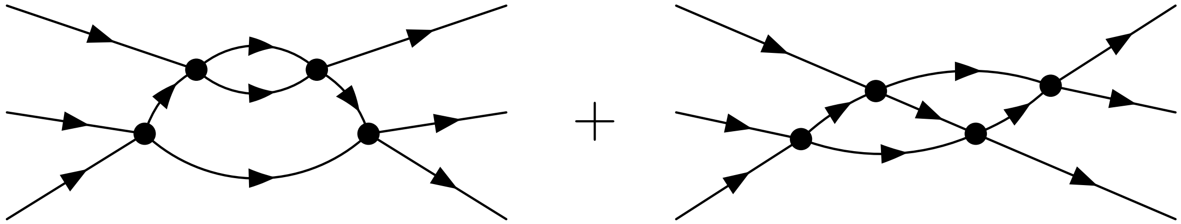
   $*$ use the separation of scales $\Longrightarrow \frac{k_{\rm F}}{\Lambda}$ with $\Lambda \sim 1/R \Longrightarrow k_{\rm F} a_s$, etc.

$\mathcal{O}(k_{\rm F}^6)$ :  $\mathcal{O}(k_{\rm F}^7)$ : 

$$\mathcal{E} = \rho \frac{k_{\rm F}^2}{2M} \left[ \frac{3}{5} + \frac{2}{3\pi}(k_{\rm F} a_s) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_{\rm F} a_s)^2 + \cdots \right]$$

## Inevitability of Three-Body Interactions $\Longrightarrow$ Analytic Structure

- 3–3 scattering $\Longrightarrow$ new log divergences (dim. reg. here):



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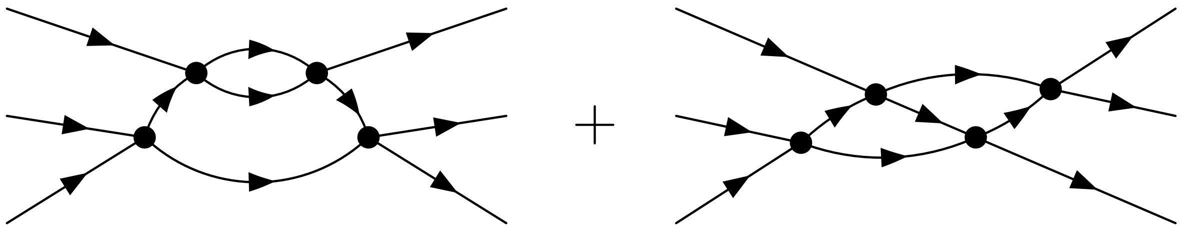

$$\mathcal{T}^{\ln}_{3\to 3} = -iM^3 (C_0)^4 \, \frac{4\pi - 3\sqrt{3}}{8\pi^3} \left[ \frac{1}{D-3} - 2 \ln \mu + \cdots \right]$$

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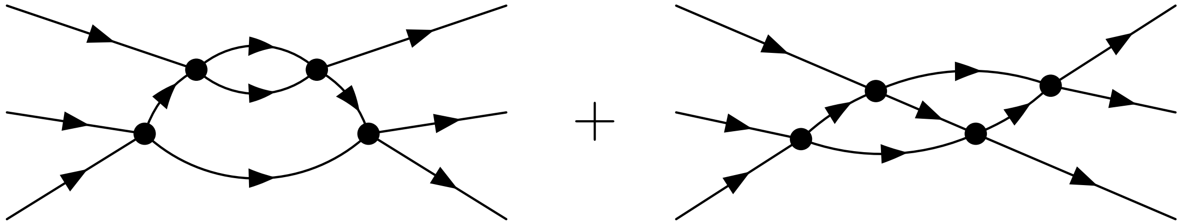

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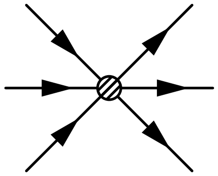

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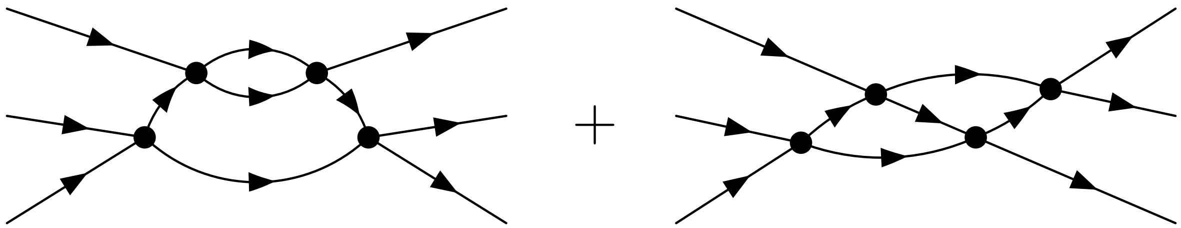

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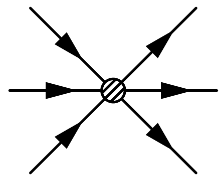


$$\Longrightarrow \qquad \mu \frac{d}{d\mu} D_0 = M^3 (C_0)^4 \, \frac{4\pi - 3\sqrt{3}}{4\pi^3}$$

which is easily solved for the $\mu$ dependence of $D_0$:

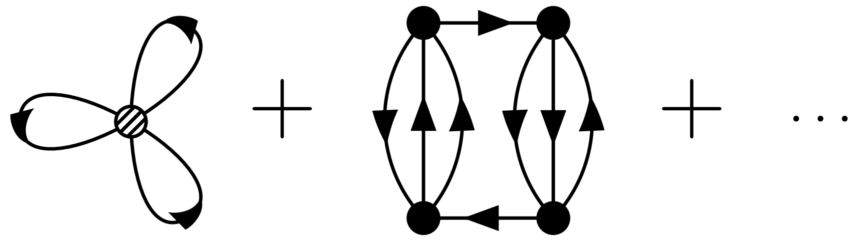
$$D_0(\mu) = D_0(1/a_s) + M^3 (C_0)^4 \, \frac{4\pi - 3\sqrt{3}}{4\pi^3} \ln(a_s \mu) \, ,$$

## Analytic Structure of the Energy Density [Hammer, rjf]

- Adapted for fermions from Braaten-Nieto boson treatment [hep-th/9609047]

- $\ln(\mu)$ dependence in energy from $D_0(\mu)$ *must* be cancelled $\Longrightarrow \ln(k_{\rm F} a_s)$!

$\mathcal{O}\big(k_{\rm F}^9 \ln(k_{\rm F})\big)$ :



- To find the log term in the energy, evaluate $D_0$ term at $\mu = k_{\rm F}$:

$$\mathcal{E}_4^{\ln} = \rho\,(g-2)(g-1)\,\frac{k_{\rm F}^2}{2M}\,\frac{16}{27\pi^3}\,\left(4\pi - 3\sqrt{3}\right)\,(k_{\rm F} a_s)^4 \ln(k_{\rm F} a_s)$$

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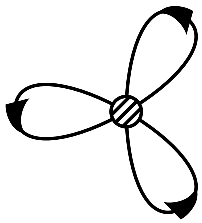
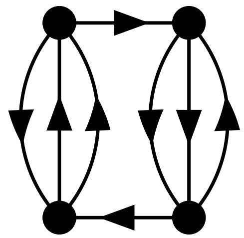
$\mathcal{O}\big(k_{\rm F}^9 \ln(k_{\rm F})\big)$ :  $+$  $+ \quad \cdots$

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- Logarithm is determined, but 3–body data required to determine $D_0(1/a_s)$

  $\Longrightarrow$ two-body data alone is insufficient

# Finding All the Logs [Braaten, Nieto]

- General structure of renormalization group equation: $\mu\frac{d}{d\mu}g_j(\mu) = \beta_j(g)$
  - $*$ matching $\Lambda$ dimensions is very restrictive! ($\mu$ only appears as $\ln\mu$):

$$C_{2i} \sim \frac{4\pi}{M\Lambda^{2i+1}}\,, \quad D_{2i} \sim \frac{4\pi}{M\Lambda^{2i+4}}$$

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- E.g., for two-body, no-derivative coefficient $C_0(\mu)$:

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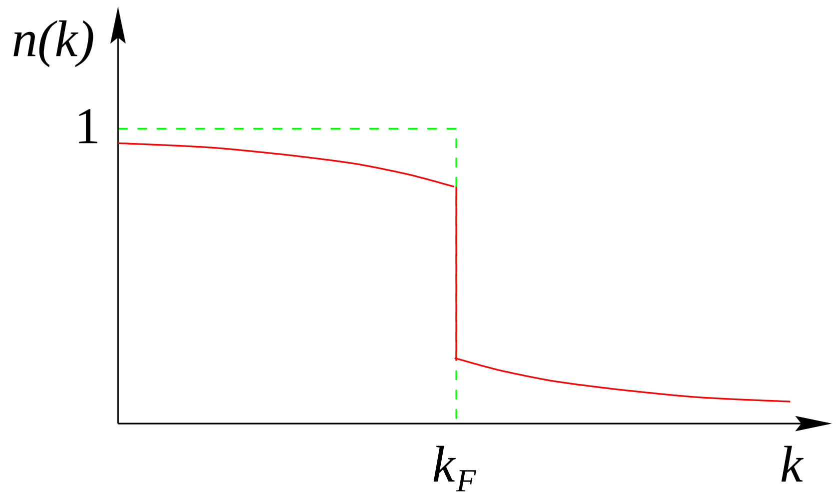
- If $\beta_j(g) \propto \ln\mu$, then $(\ln\mu)^2$, and so on

# Are Occupation Numbers Observable? [Hammer, Tirfessa, rjf]

- Uniform system with $\widehat{H} = \sum_{rs} a_r^\dagger \langle r|T|s\rangle a_s + \frac{1}{2}\sum_{rstu} a_r^\dagger a_s^\dagger \langle rs|V|tu\rangle a_u a_t$
  - $\ast$ $\hat{n}_k \equiv a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \Longrightarrow n(k) = \langle \hat{n}_k \rangle$ is momentum distribution
  - $\ast$ forward scattering (direct/exch.) only $\Longrightarrow n(k) = \theta(k_{\rm F} - k)$

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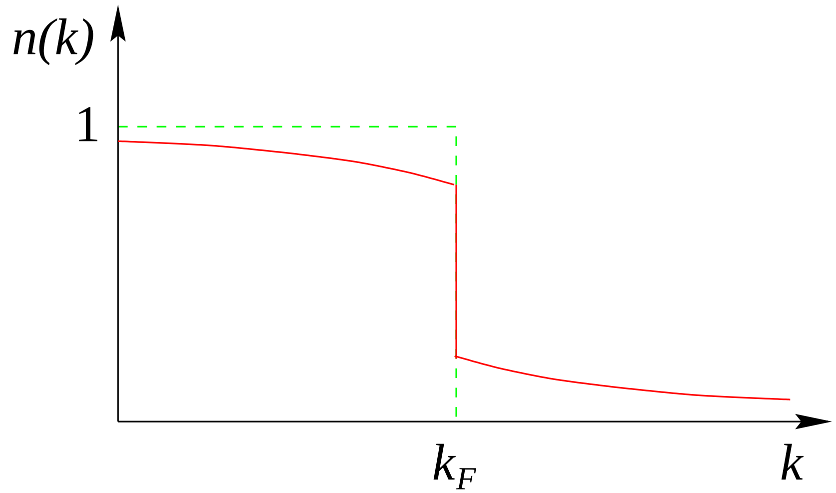



  - $\ast$ fermions scatter $> k_{\rm F} \Longrightarrow$ depletion $< k_{\rm F}$, non-zero $> k_{\rm F}$
  - $\ast$ depletion as a measure of strength of correlations
- Claim: one can measure $n(k)$ in $(e, e'p)$ on a nucleus
- Here: Is $n(k)$ an observable or model dependent? What does EFT say?

# Field Redefinitions at Finite Density

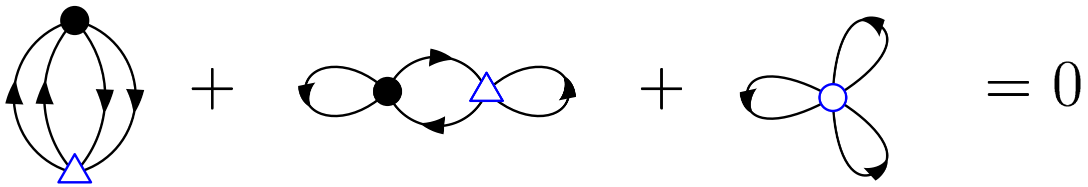
- Consider simple field redefinition to find model dependence:

$$\psi \longrightarrow \psi + \alpha \frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi \qquad \alpha \sim \mathcal{O}(1) \Longrightarrow \text{“natural”}$$

  $*$ vertices induced: 2–body off-shell △ and 3–body ○ $\propto \frac{8\pi\alpha}{\Lambda^3} C_0 (\psi^\dagger\psi)^3$

  $*$ asymptotic “on-shell” quantities (S-matrix elements) unchanged

- Energy density is model ($\alpha$) independent *if* all terms kept (“Coester line”)



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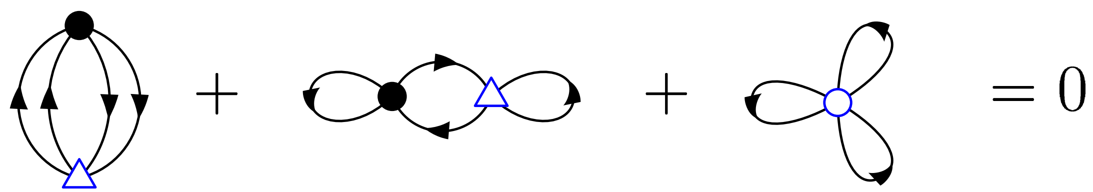

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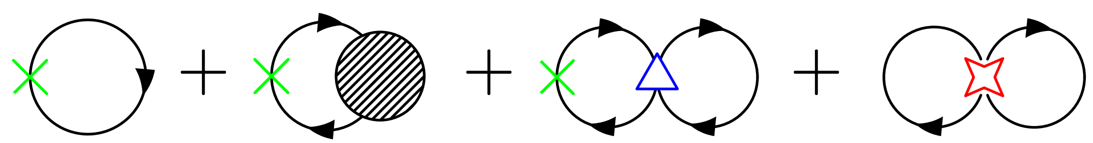

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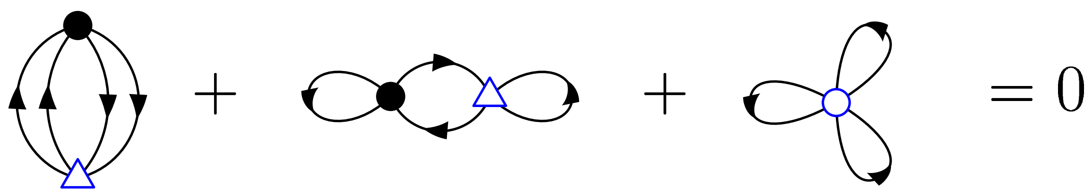


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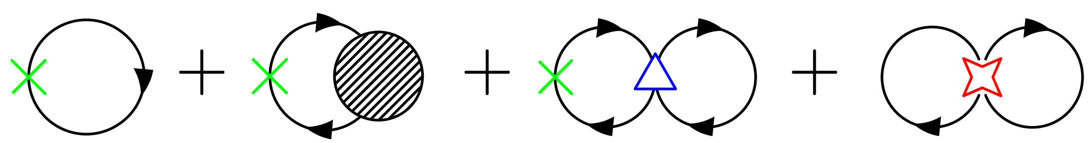
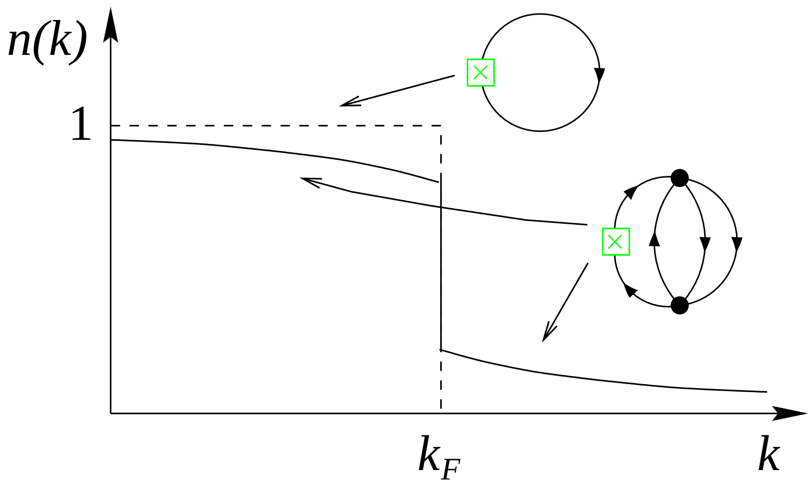



- What about momentum occupation number?

# Occupation No. $\Longrightarrow$ Momentum Distribution

- Insert $a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \Longrightarrow \boxtimes$



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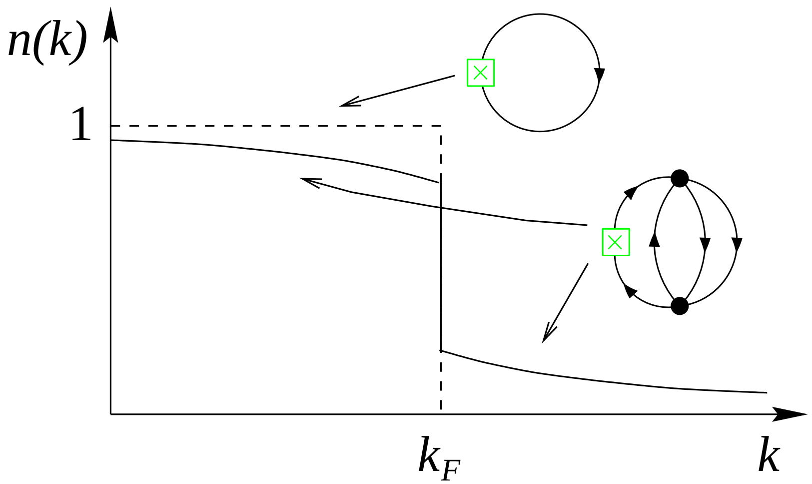


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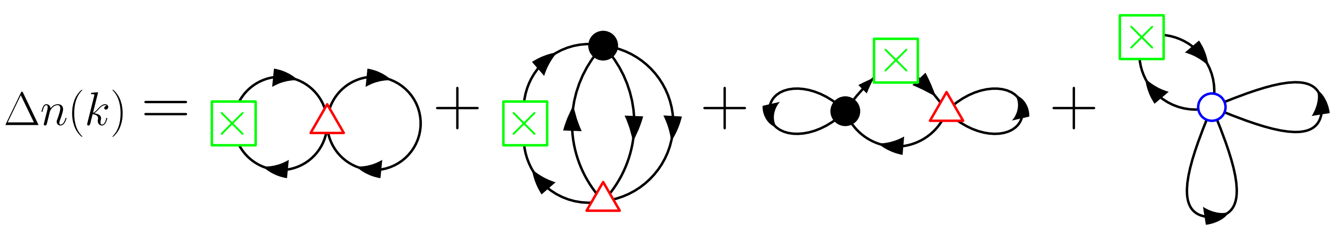

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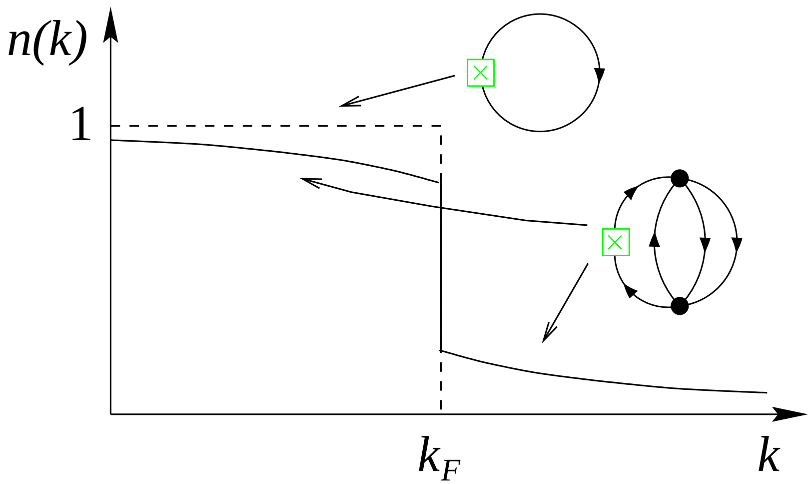


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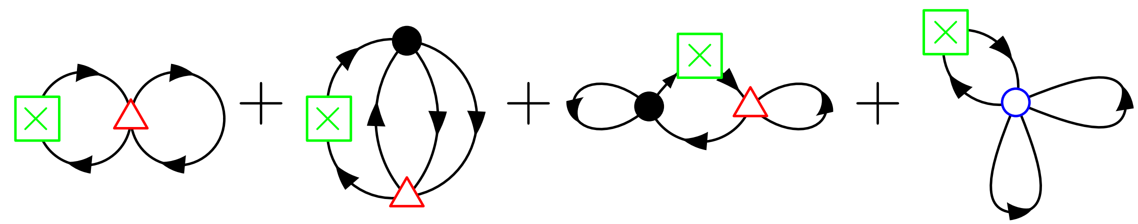

- No preferred definition for transformed operator

  $\Longrightarrow$ only defined in specific convention

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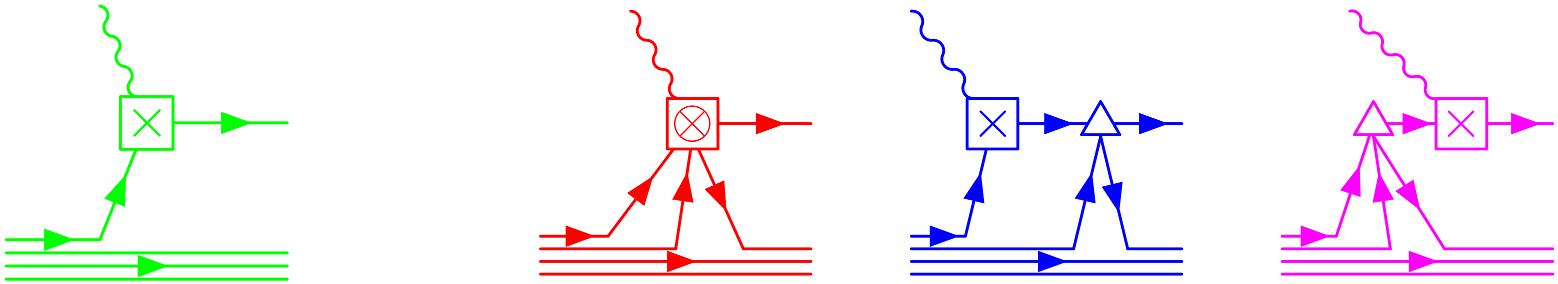

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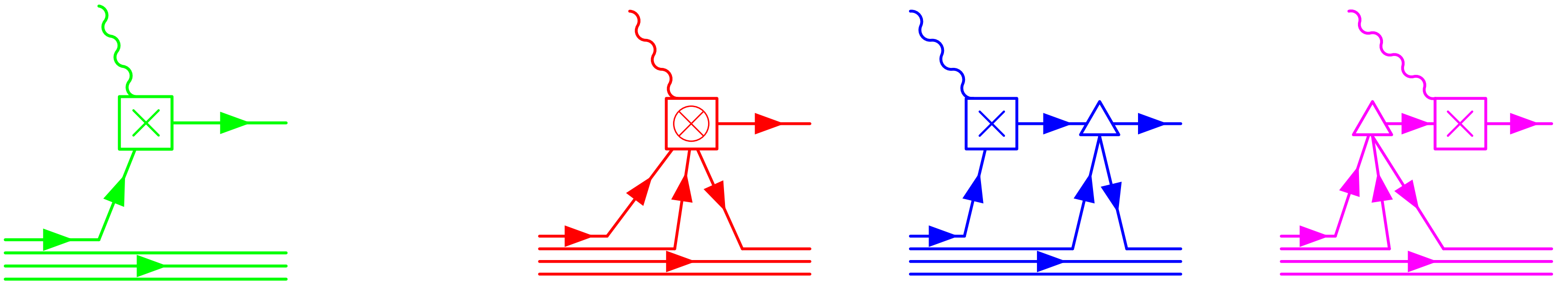

  - $*$ if $\alpha \neq 0$ then same cross section *only* if vertex contribution from modified operator *and* induced final (and initial) state interactions
- There are *always* contributions of all three types in each order
  - $*$ mixed by field redefinitions $\Longrightarrow$ isolating $J\psi^\dagger\psi$ is model-dependent

# Occupation Numbers $\Longrightarrow$ Comments

- Experiment cannot resolve ambiguities in momentum distributions
  - $*$ they cannot be isolated in principle from $(e, e'p)$ within a calculational framework based on low-energy degrees of freedom
  - $*$ auxiliary quantities defined only in a specific convention $\Longrightarrow$ still useful within convention but not an observable!
  - $*$ ambiguities have a natural size (which may be negligible!)
- Analogous situations:
  - $*$ deep inelastic scattering: physical cross section from [quark and gluon distributions] $*$ [coefficient functions]
    - ○ individually scheme and scale dependent
    - ○ also pion distribution in nucleon [Chen and Ji, hep-ph/0107158]
  - $*$ condensate fraction in a Bose-Einstein condensate?

# Some Current Trends in Many-Body EFT

- Non-perturbative effective action formalism
  - ∗ large N expansion [Hammer and rjf, nucl-th/0208058]
  - ∗ in progress: large scattering length problem
    - ○ expansion in 1/space-time dimension revisited [Steele]
- Application to finite systems $\Longrightarrow$ density functional theory
  - ∗ dilute Fermi system [Puglia, Bhattacharyya, and rjf, nucl-th/0212071]
  - ∗ in progress: long-range forces, pairing, systematic gradient expansion
- Future applications
  - ∗ energy functionals for nuclei far from stability
  - ∗ trapped fermionic atoms and superfluidity