Similarity Renormalization Group Evolution of Many-Body Forces in a One-Dimensional Model

E.D. Jurgenson

Department of Physics
The Ohio State University

APS Division of Nuclear Physics
Oakland, CA
October 23-26, 2008

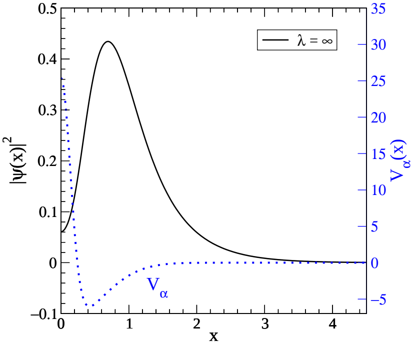
Work supported by NSF and UNDEF/SciDAC (DOE)
Collaborators: E.R. Anderson, S.K. Bogner, R.J. Furnstahl, R.J. Perry

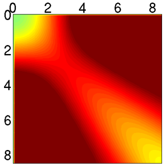
E.D.J., R.J. Furnstahl [arXiv:0809.4199]

SRG Review - Series of Unitary Transformations

$$\frac{dH_s}{ds} = U_s H U_s^\dagger = [[G_s, H_s], H_s] \quad G_s \Rightarrow T \quad (s \equiv 1/\lambda^4)$$

$$\text{1-D model: } V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$

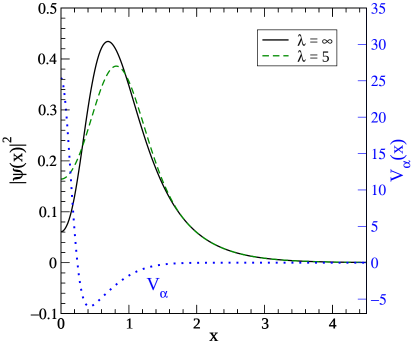


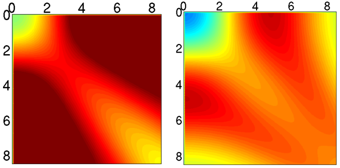


- $\frac{dV^{(2)}}{ds} + \frac{dV^{(3)}}{ds} + \ldots = [G_s, (V_s^{(2)} + V_s^{(3)} + \ldots)], (V_s^{(2)} + V_s^{(3)} + \ldots)]$
- How do we handle many-body forces?

$$\frac{dH_s}{ds} = U_s H U_s^\dagger = [[G_s, H_s], H_s] \quad G_s \Rightarrow T \quad (s \equiv 1/\lambda^4)$$

$$\text{1-D model:} \;\; V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$

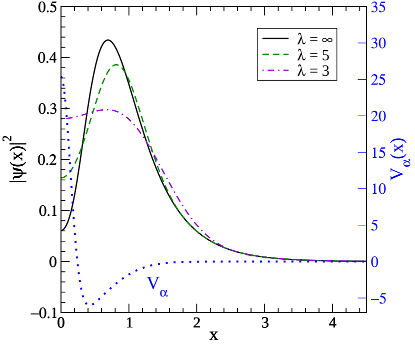


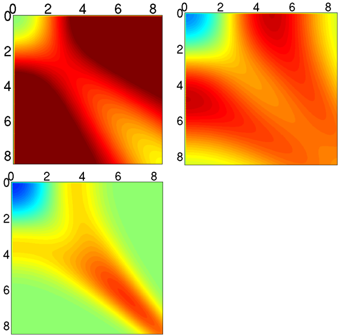


- $\frac{dV^{(2)}}{ds} + \frac{dV^{(3)}}{ds} + \ldots = [G_s, (V_s^{(2)} + V_s^{(3)} + \ldots)], (V_s^{(2)} + V_s^{(3)} + \ldots)]$
- How do we handle many-body forces?

$$\frac{dH_s}{ds} = U_s H U_s^\dagger = [[G_s, H_s], H_s] \quad G_s \Rightarrow T \quad (s \equiv 1/\lambda^4)$$

1-D model: $V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$



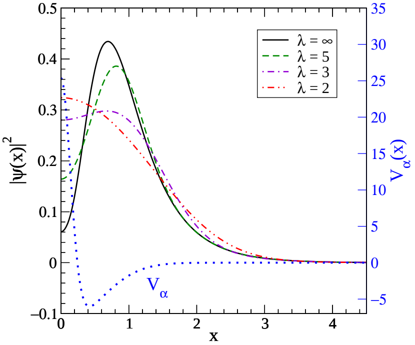


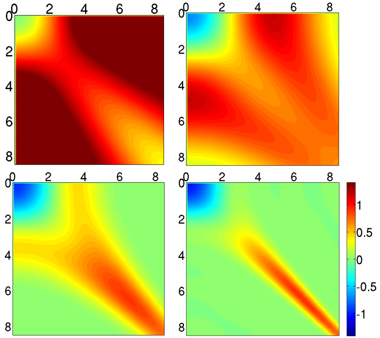
- $\frac{dV^{(2)}}{ds} + \frac{dV^{(3)}}{ds} + \ldots = [G_s, (V_s^{(2)} + V_s^{(3)} + \ldots)], (V_s^{(2)} + V_s^{(3)} + \ldots)]$
- How do we handle many-body forces?

SRG Review - Series of Unitary Transformations

$$\frac{dH_s}{ds} = U_s H U_s^\dagger = [[G_s, H_s], H_s] \quad G_s \Rightarrow T \quad (s \equiv 1/\lambda^4)$$

1-D model: $V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$

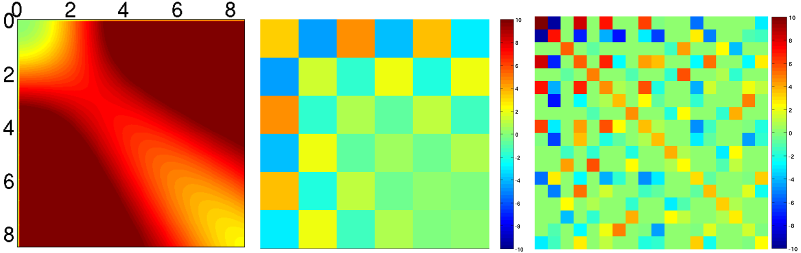




- $\frac{dV^{(2)}}{ds} + \frac{dV^{(3)}}{ds} + \ldots = [G_s, (V_s^{(2)} + V_s^{(3)} + \ldots)], (V_s^{(2)} + V_s^{(3)} + \ldots)]$
- How do we handle many-body forces?

Embedding: initial potential

- Discrete basis avoids "dangerous" k-space delta functions
- Symmetrized Jacobi Oscillator Basis (here: Bosons)

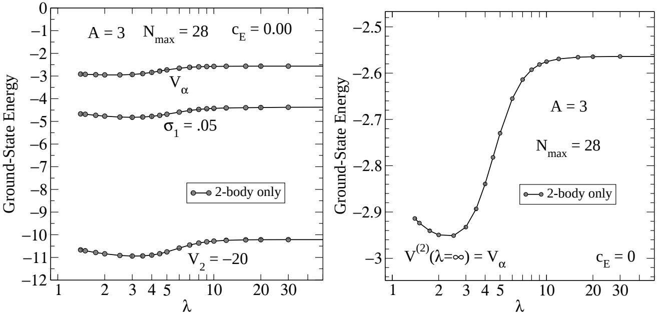


$$V(p,p') \longrightarrow V(N_2,N_2') \longrightarrow V(N_3,N_3')$$

- diagonalize symmetrizer $\Rightarrow \langle N_A || N_{A-1}; n_{A-1} \rangle$; use recursively
- 3D: Use Navratil et al. technology for NCSM
- embedding is everything, SRG is trivial

Induced Many-Body Forces are Small - A=3

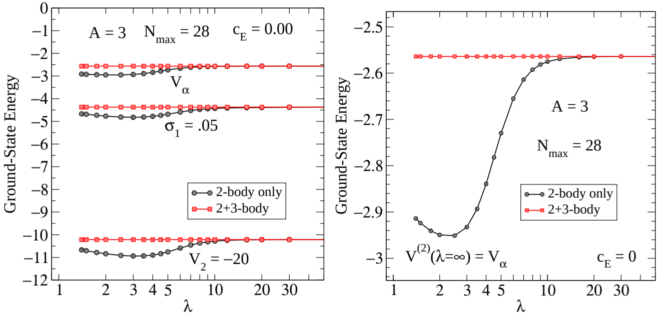
$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$



- Basis independent: same evolution in momentum or HO basis
- Red: unitary, Black: 2-body only ≈ unitary
- Same evolution pattern for 2-body only as 3D NN

Induced Many-Body Forces are Small - A=3

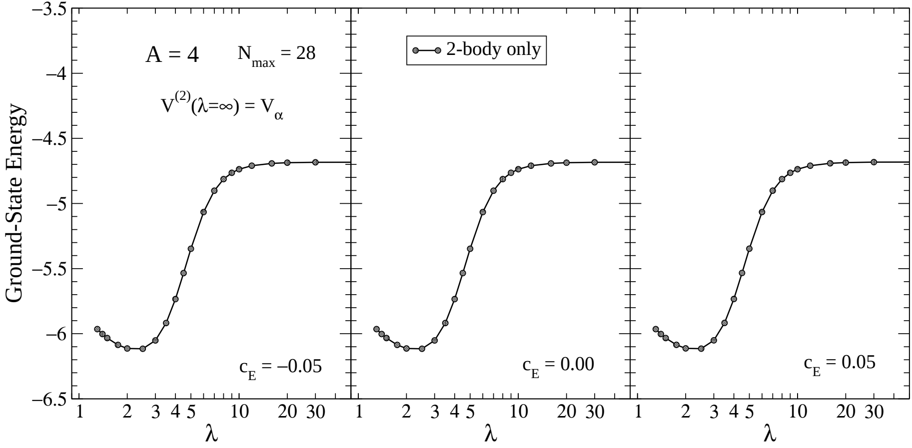
$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$



- Basis independent: same evolution in momentum or HO basis
- Red: unitary, Black: 2-body only ≈ unitary
- Same evolution pattern for 2-body only as 3D NN

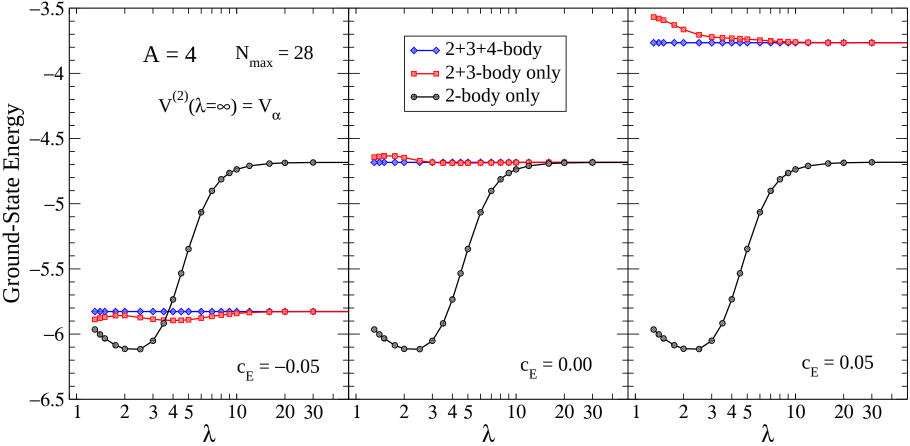
Induced Many-Body Forces are Small - A=4

$$V^{(3)}(p,q,p',q') = c_E e^{-((p'^2+q'^2)/\Lambda^2)^n} e^{-((p^2+q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4)$$

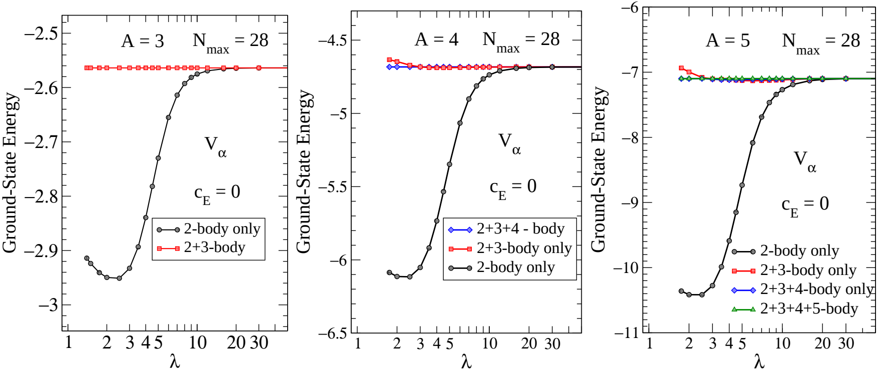


Induced Many-Body Forces are Small - A=4

$$V^{(3)}(p,q,p',q') = c_E e^{-((p'^2+q'^2)/\Lambda^2)^n} e^{-((p^2+q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4)$$



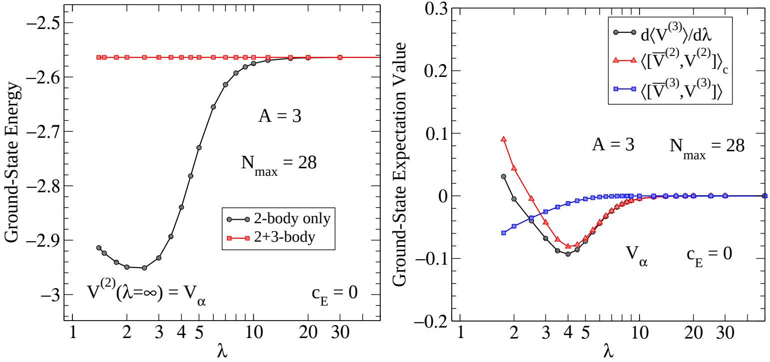
Induced Many-Body Forces are Small - A=5



- Five-body force is negligible
- Hierarchy of induced many-body forces

$V^{(3)}$ analysis

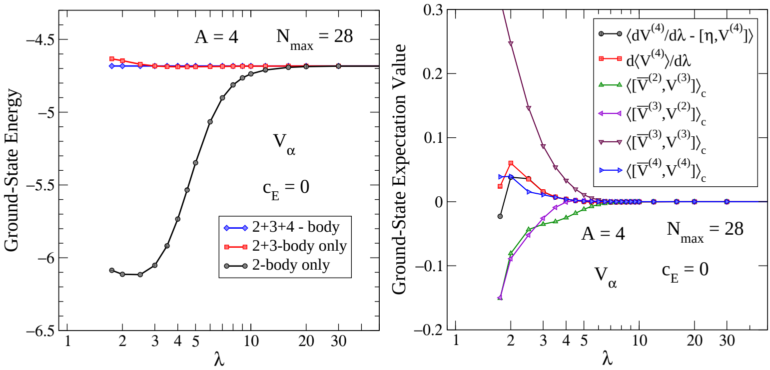
$$\frac{d}{d\lambda}\langle\psi_\lambda^{(3)}|V_\lambda^{(3)}|\psi_\lambda^{(3)}\rangle = \langle\psi_\lambda^{(3)}|[\overline{V}_\lambda^{(2)}, V_\lambda^{(2)}]_c - [\overline{V}_\lambda^{(3)}, V_\lambda^{(3)}]|\psi_\lambda^{(3)}\rangle$$



- Majority evolution dominated by $[\overline{V}^{(2)}, V^{(2)}]$, $(\overline{V} \equiv [T, V])$
- Hierarchy of contributions

$V^{(4)}$ analysis

$$\frac{d}{d\lambda}\langle\psi_\lambda^{(4)}|V_\lambda^{(4)}|\psi_\lambda^{(4)}\rangle = \langle\psi_\lambda^{(4)}|[\overline{V}_\lambda^{(2)}, V_\lambda^{(3)}]_c + [\overline{V}_\lambda^{(3)}, V_\lambda^{(2)}]_c + [\overline{V}_\lambda^{(3)}, V_\lambda^{(3)}]_c - [\overline{V}_\lambda^{(4)}, V_\lambda^{(4)}]|\psi_\lambda^{(4)}\rangle$$



- No $[\overline{V}^{(2)}, V^{(2)}]$, Hierarchy persists
- Induced 4-body is small

Conclusions

Recap

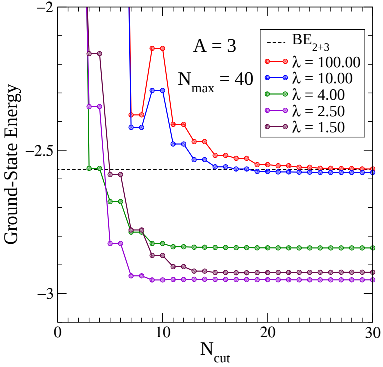
- Induced Many-Body forces maintain a hierarchy
- 3D generalizations straightforward (Do findings persist?)

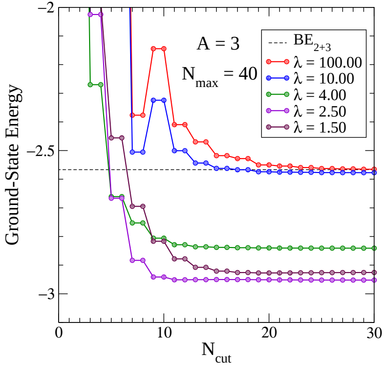
Outlook for 1D Model

- Labframe version
- Individual Operator evolutions (see next talk)
- Different SRG choices
- In-medium SRG

Bonus Slides

SRG is basis independent





SRG basis: Momentum Oscillator

Lab Frame

$$H_{eff} = T + V_{ij} + V_{ho} + (\beta - 1)H_{ho} - \beta\frac{\hbar\omega}{2}$$

- Use H_{tot} in a mean field harmonic oscillator
- Boost other COM solutions
- Check on Jacobi Calculations
- Easier to generalize
- More computationally intensive

Operator Evolution

show the evolving wavefunction with the bare operator