Similarity Renormalization Group for Few-Body Systems

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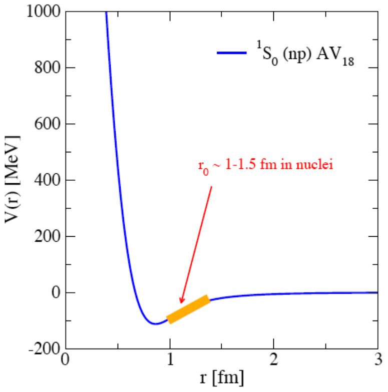
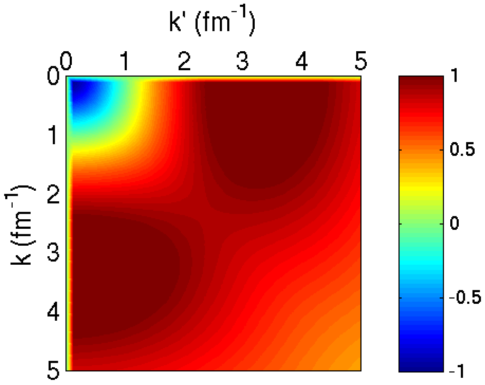
SRG: S. Glazek, K. Wilson, PRD **48** (1993) 5863; **49** (1994) 4214;
F. Wegner, Ann. Phys. **3** (1994) 77.

Collaborators: E. Anderson, S. Bogner, S. Glazek, E. Jurgenson, P. Maris, R. Perry, S. Ramanan, A. Schwenk, J. Vary

Sources of Nonperturbative Physics for NN





1 Strong short-range repulsion (“hard core” or singular $V_{2\pi}$)

2 Iterated tensor (S_{12}) interaction

3 Near zero-energy bound states

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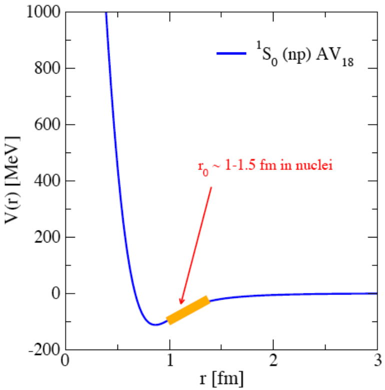
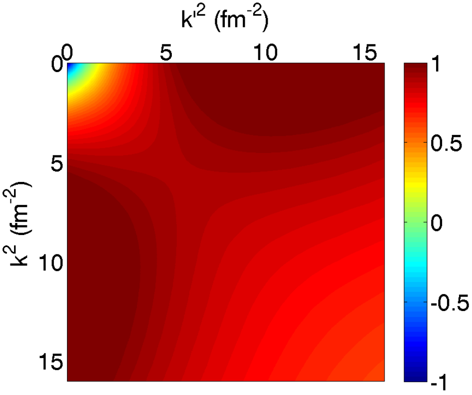
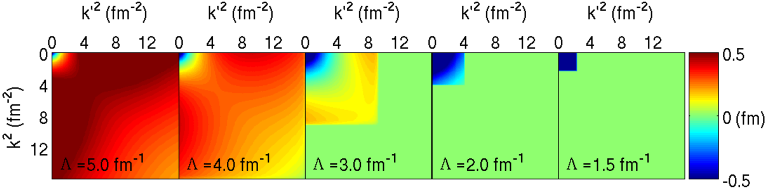




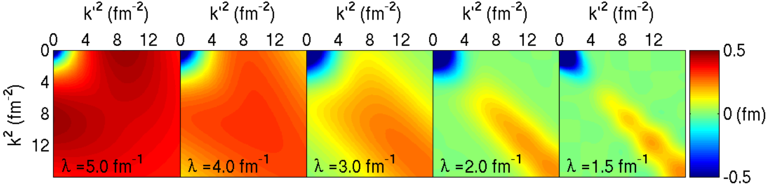
1. Strong short-range repulsion (“hard core” or singular $V_{2\pi}$)
2. Iterated tensor (S_{12}) interaction
3. Near zero-energy bound states

Low-Momentum Interactions from RG [AV18 $^3S_1$]

- "$V_{\mathrm{low}\,k}$" $\Longrightarrow$ Lower a cutoff Λ in relative k, k' [sharp]



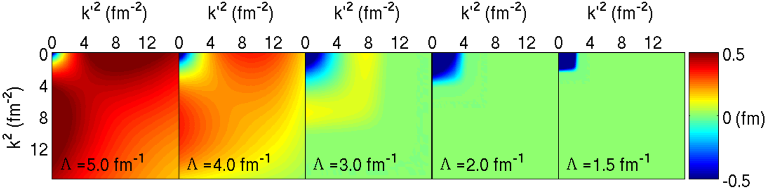
- SRG $\Longrightarrow$ Drive the Hamiltonian toward diagonal [$\lambda \equiv 1/s^{1/4}$]



- Other transformations also decouple (e.g., UCOM)
- Isn't chiral EFT already soft? Or why not use a lower cutoff? [e.g., E/G/M: 450 MeV, E/M: N3LOW (400 MeV)]

Low-Momentum Interactions from RG [AV18 $^3S_1$]

- "$V_{\text{low }k}$" $\Longrightarrow$ Lower a cutoff Λ in relative k, k' [$e^{-(k^2/\Lambda^2)^8}$]



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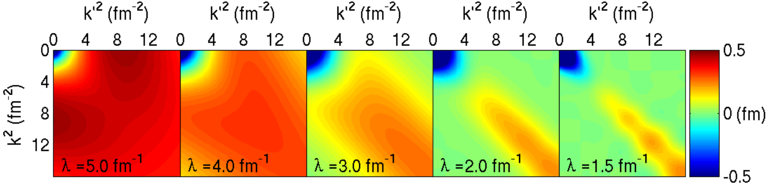

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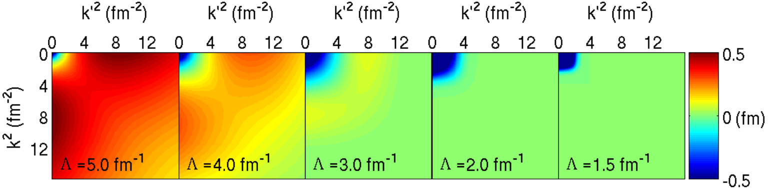

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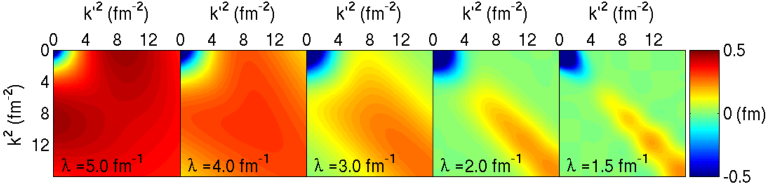

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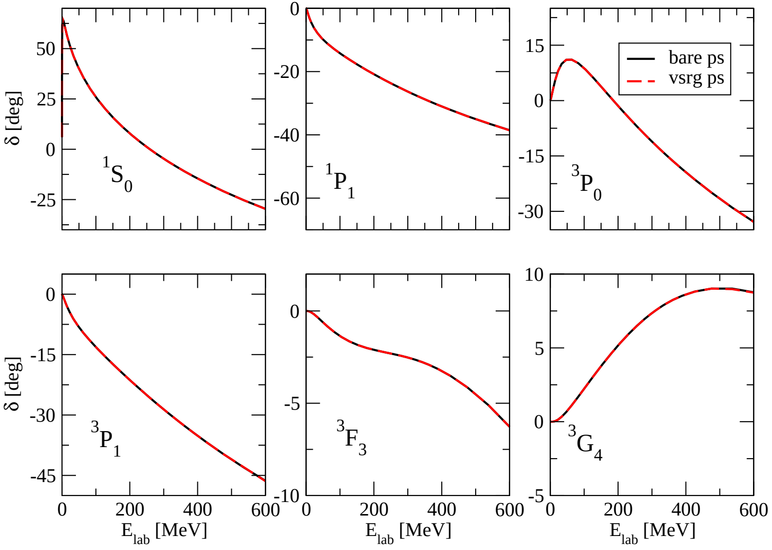
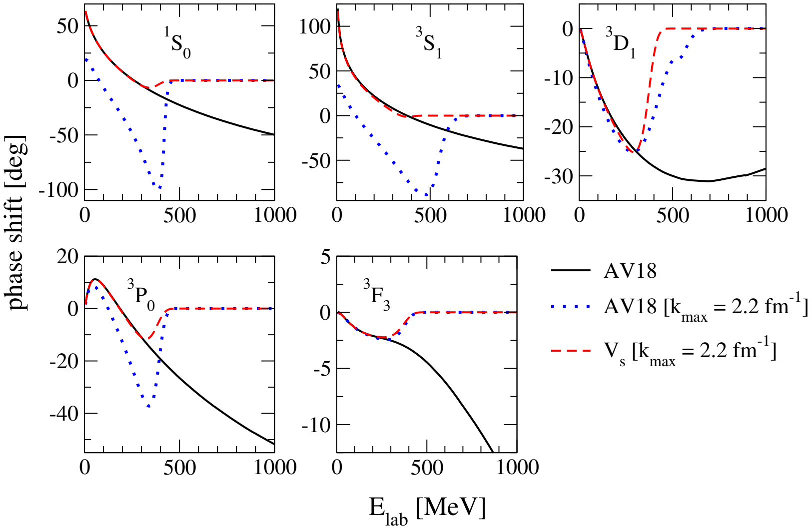
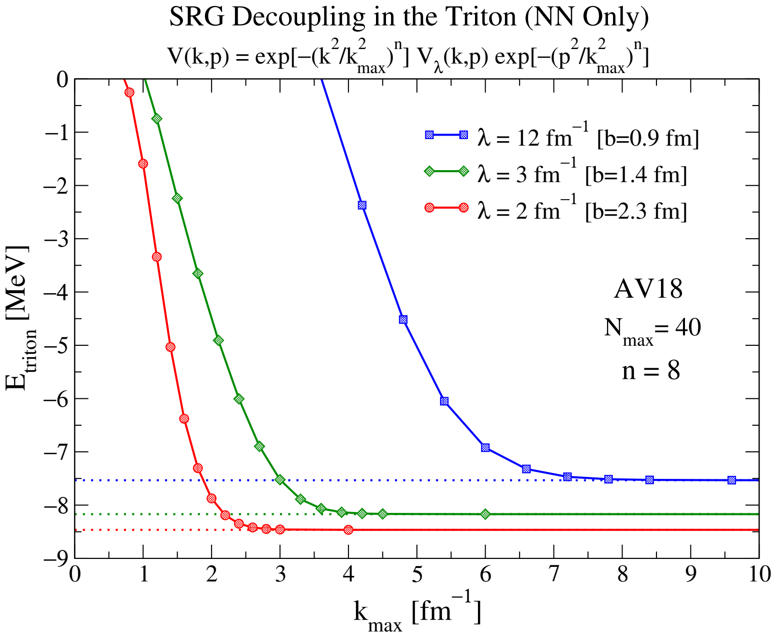
Unitary Transformation: Bare vs. SRG phase shifts



Decoupling Demonstrated [nucl-th/0701013]

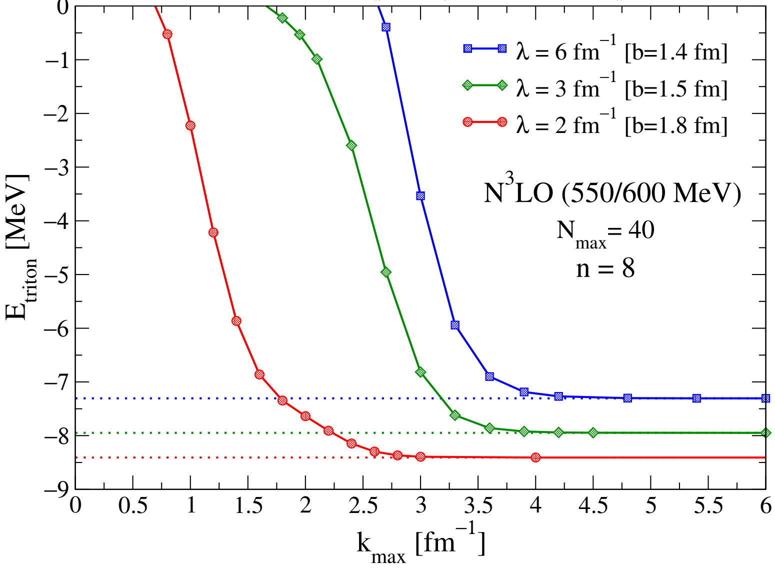
- Phase shifts with $V_s(k,k') = 0$ for $k, k' > k_{\text{max}}$




SRG Decoupling in the Triton (NN Only)
V(k,p) = exp[−(k²/k²max)ⁿ] Vλ(k,p) exp[−(p²/k²max)ⁿ]
λ = 12 fm⁻¹ [b=0.9 fm]
λ = 3 fm⁻¹ [b=1.4 fm]
λ = 2 fm⁻¹ [b=2.3 fm]
AV18
Nmax = 40
n = 8
Etriton [MeV]
kmax [fm⁻¹]
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
0 1 2 3 4 5 6 7 8 9 10

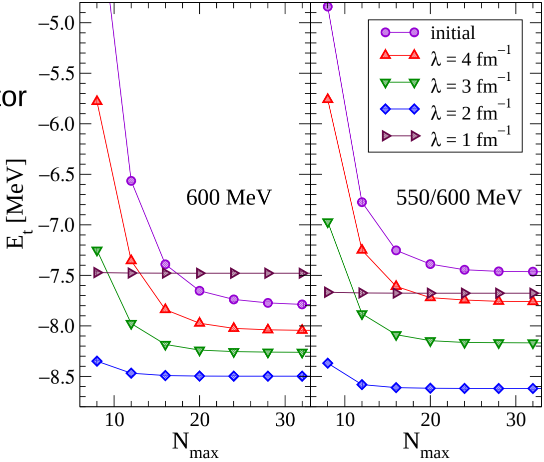
SRG Decoupling in the Triton (NN Only)

$$V(k,p) = \exp[-(k^2/k_{max}^2)^n]\, V_\lambda(k,p)\, \exp[-(p^2/k_{max}^2)^n]$$



Variational Calculations Converge More Rapidly

- Triton ground-state energy vs. size of harmonic oscillator basis ($N_{\max}\hbar\omega$ excitations)
- Rapid convergence as λ decreases
- See arXiv:0708.3754, S. Bogner talk (Session 4A) for more examples!
- Different binding energies $\Longrightarrow$ 3-body contribution



Band Diagonalizing with SRG [nucl-th/0611045]

- Transform an initial hamiltonian, $H = T + V$:

$$H_s = U(s) H U^\dagger(s) \equiv T + V_s \ ,$$

where s is the *flow parameter*. Differentiating wrt s:

$$\frac{dH_s}{ds} = [\eta(s), H_s] \qquad \text{with} \qquad \eta(s) \equiv \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s) \ .$$

- $\eta(s)$ is specified by the commutator with "T_{diagonal}" $\Longrightarrow T_D$:

$$\eta(s) = [T_D, H_s] \ ,$$

which yields the flow equation,

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[T_D, H_s], H_s] \ .$$

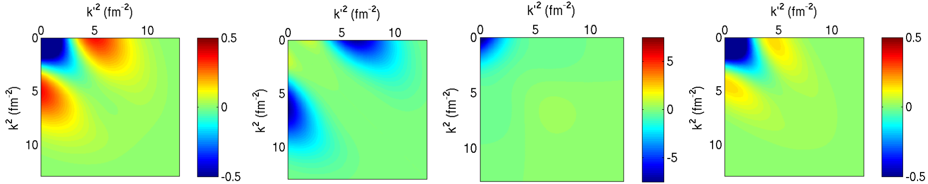
- T_D determines flow: $T_D = T$ or T^2 or $H_D (\equiv T + (V_s)_D)$ or ...

Flow in NN Momentum Basis with $\eta(s) = [T, H_s]$

- For NN only, project onto partial-wave rel. momentum $|k\rangle$

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \qquad \text{with} \quad \epsilon_k = \hbar^2 k^2/M \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



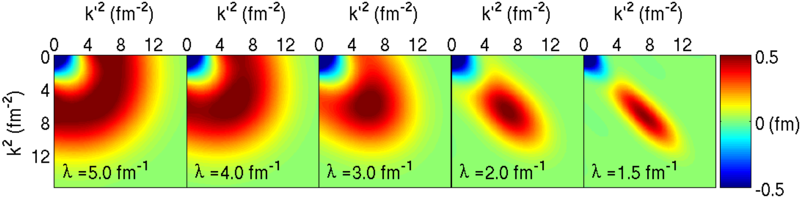
$V_{\lambda=2.5}(k,k')$ | 1st term | 2nd term | $V_{\lambda=2.0}(k,k')$

- First term drives V_λ toward diagonal:

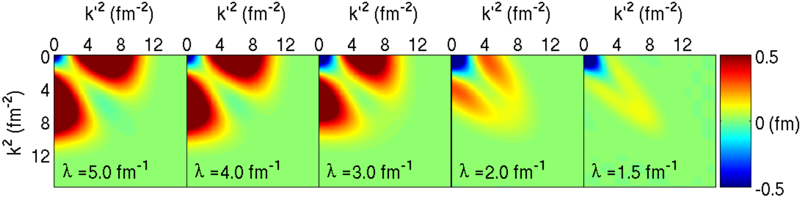
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \cdots$$

Flow of N^{3}LO Potentials

- $^1S_0$ from N^{3}LO (500 MeV) of Entem/Machleidt



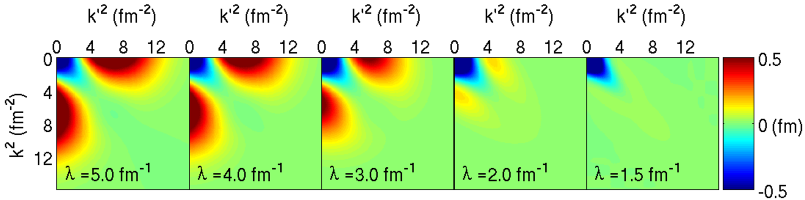
- $^1S_0$ from N^{3}LO (550/600 MeV) of Epelbaum et al.



- See http://www.physics.ohio-state.edu/∼srg/ for more!

Flow of N^3LO Potentials

- ${}^3S_1$ from N^3LO (500 MeV) of Entem/Machleidt



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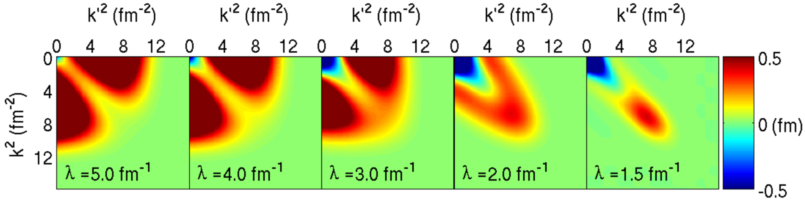

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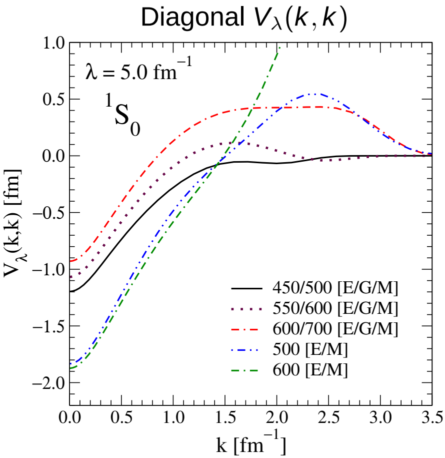
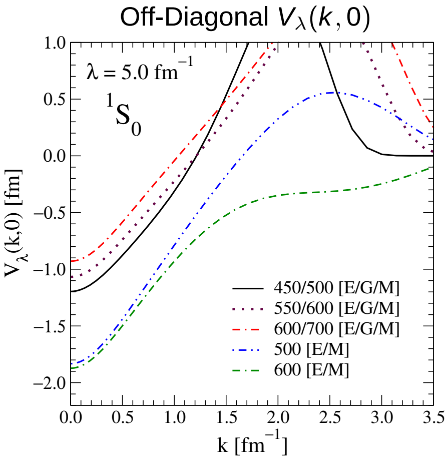
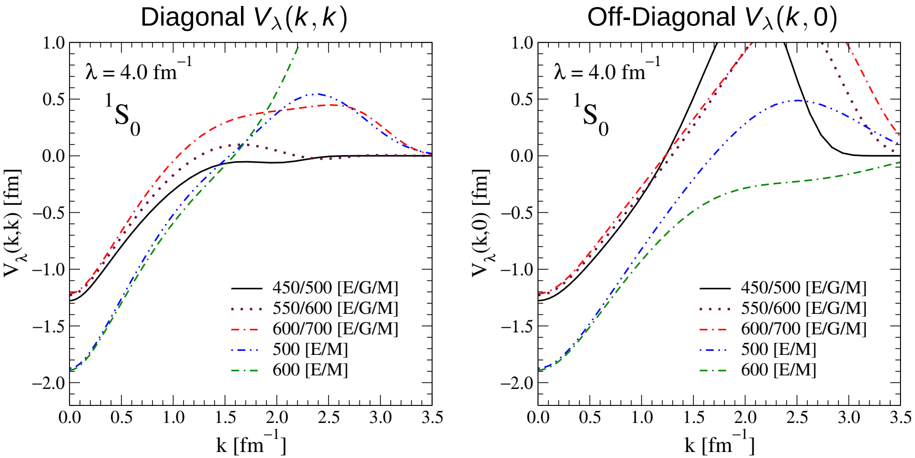
Run to Lower λ via SRG $\Longrightarrow$ ≈Universality





- Will evolved NNN interaction show universality?

Run to Lower λ via SRG $\Longrightarrow$ $\approx$Universality



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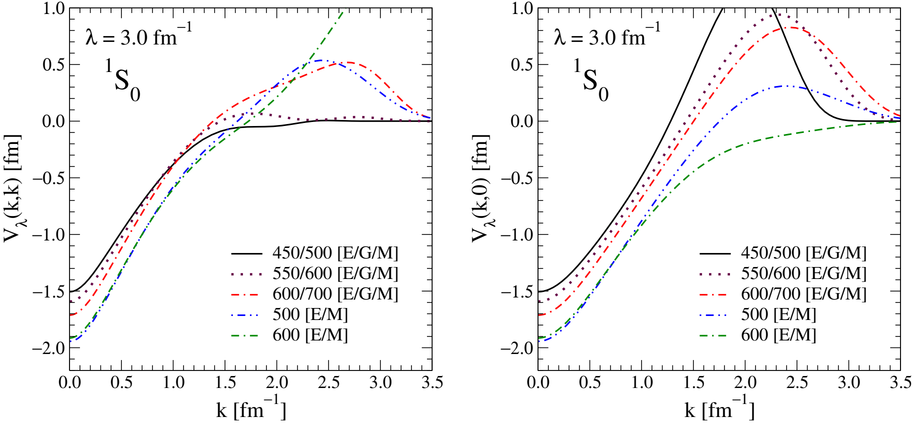
Diagonal $V_\lambda(k,k)$ Off-Diagonal $V_\lambda(k,0)$



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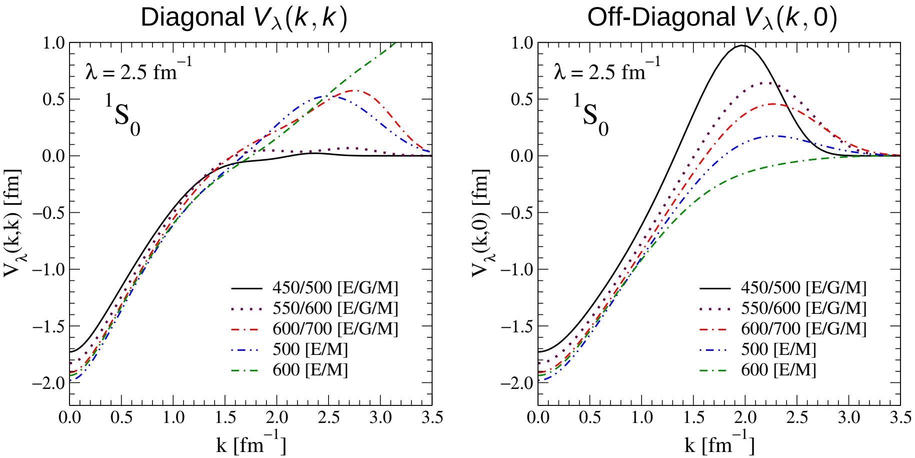

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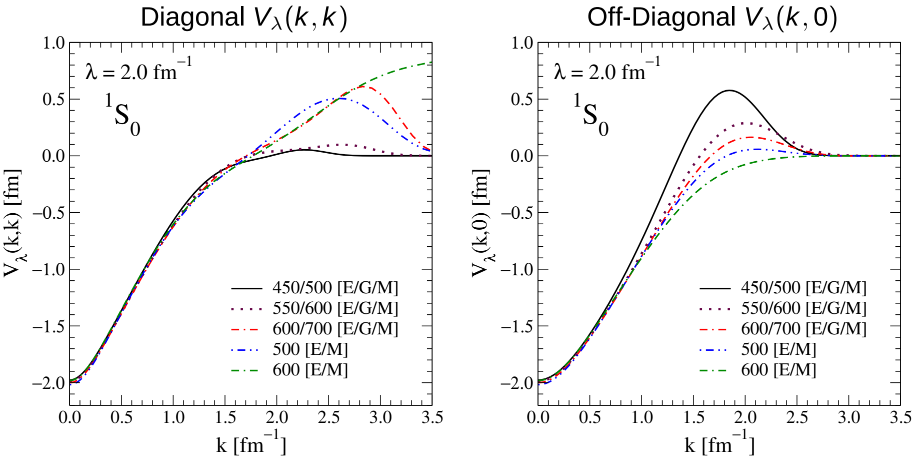

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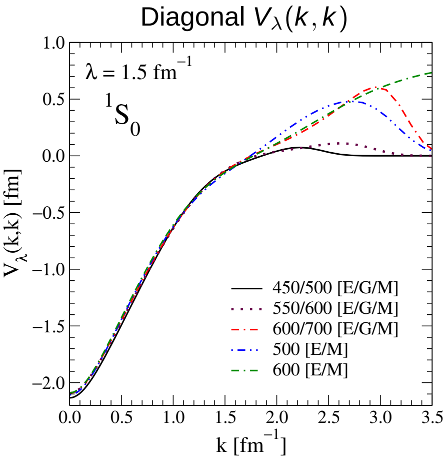
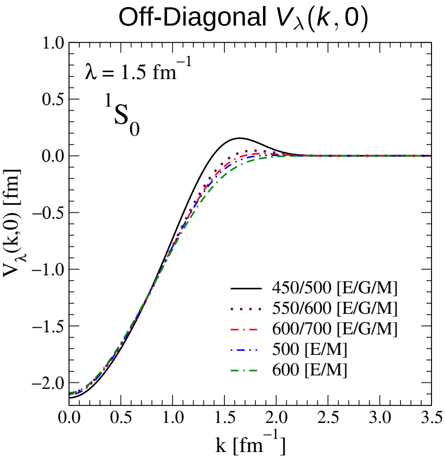
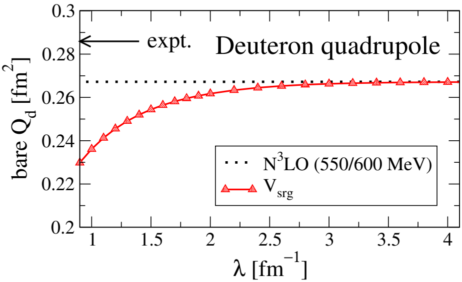


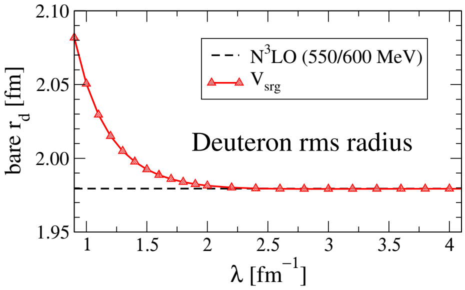


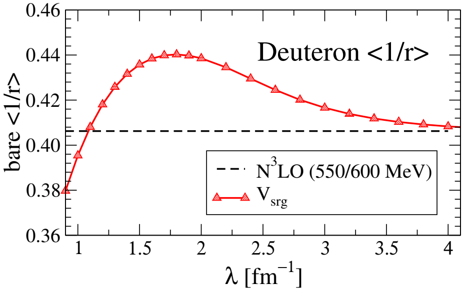
- Will evolved NNN interaction show universality?

Long-Distance Physics Preserved

- Matrix elements dominated by long range run slowly for $\lambda \gtrsim 2\,\text{fm}^{-1}$
- Here: examples from deuteron (compressed scales)







Every Operator Flows

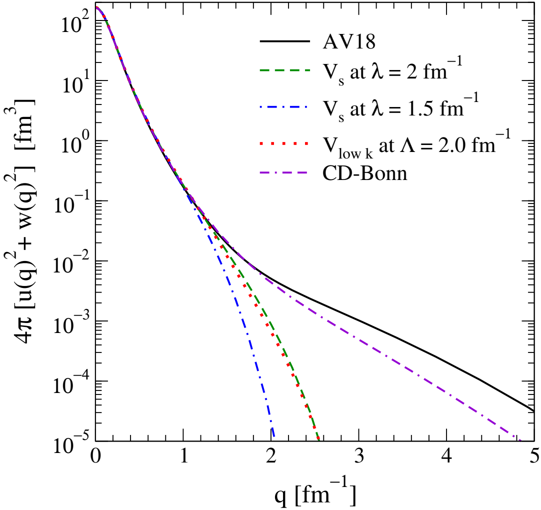
- Evolution with s of any operator O is given by:

$$O_s = U(s) O U^\dagger(s)$$

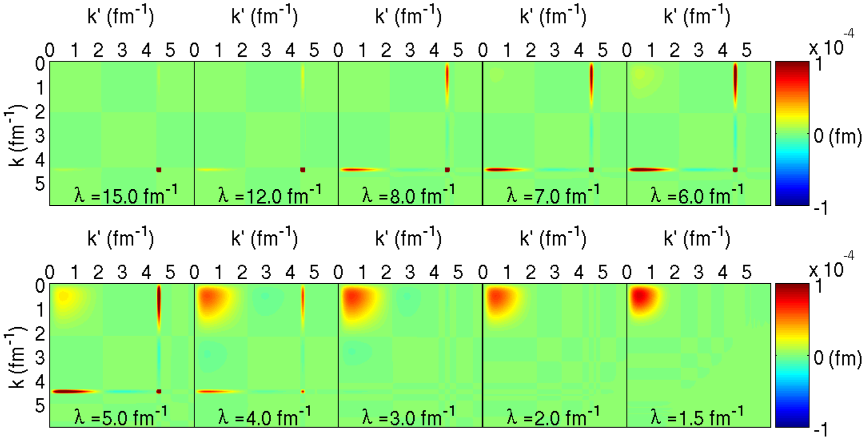
so O_s evolves via

$$\frac{dO_s}{ds} = [[T_{\rm rel}, V_s], O_s]$$

- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 4.5\,\mathrm{fm}^{-1}$



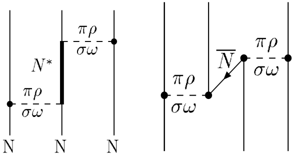
Integrand of $< \psi_d | U a_q^\dagger a_q U^\dagger | \psi_d >$ at $q = 4.5\,\text{fm}^{-1}$

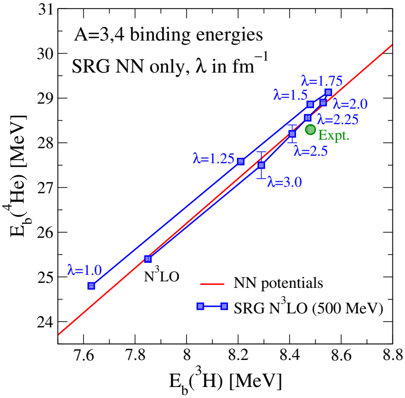


- Flow of deuteron matrix element integrand is toward low k
- Simple variational ansatz works well $\Longrightarrow$ No fine-tuning
- Factorization: $U(k,q) \longrightarrow K(k)Q(q)$ for $k \leq \lambda$, $q \gg \lambda$

Observations on Three-Body Forces

- Three-body forces arise from eliminating dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on λ
  - e.g., Tjon line
- 3-body contributions increase with density
  - saturates nuclear matter
  - how large is 4-body?





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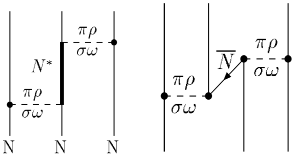
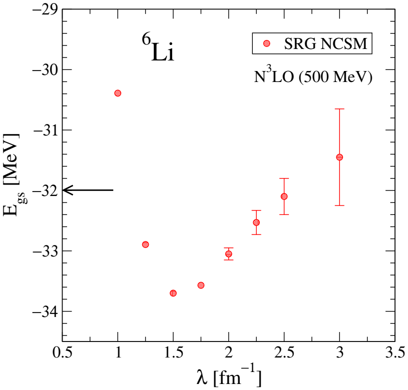


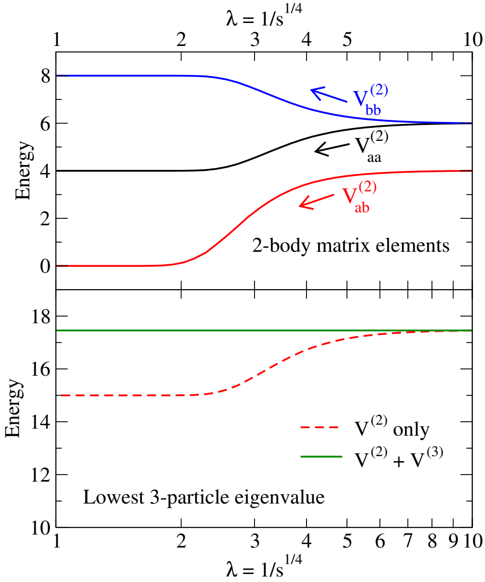


SRG 3-Body in Simple Test Cases

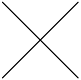
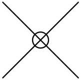
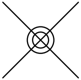
- See arXiv:0708.1502
- E.g., two-level system with 2 or 3 particles
- SRG diagonalizes

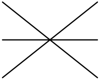
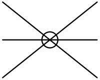
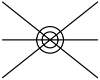
$$H^{(2)} = \begin{pmatrix} \epsilon_a + V^{(2)}_{aa} & V^{(2)}_{ab} \\ V^{(2)}_{ba} & \epsilon_b + V^{(2)}_{bb} \end{pmatrix}$$

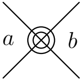
- Diagonalize *after* evolving
- Can evolve $V^{(2)} + V^{(3)}$ together here!
- 3-particle eigenvalues run if no 3-body “force”, but invariant with SRG running of $V^{(3)}$

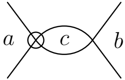


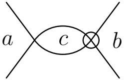
Diagrams for SRG $\Longrightarrow$ Disconnected Cancels

$V_s^{(2)} =$  $[T, V_s^{(2)}] =$  $[[T, V_s^{(2)}], T] =$ 

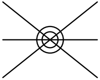
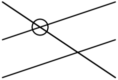
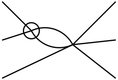
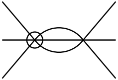
$V_s^{(3)} =$  $[T, V_s^{(3)}] =$  $[[T, V_s^{(3)}], T] =$ 

$\dfrac{dV_s^{(2)}(a,b)}{ds} =$ 

$+$ 

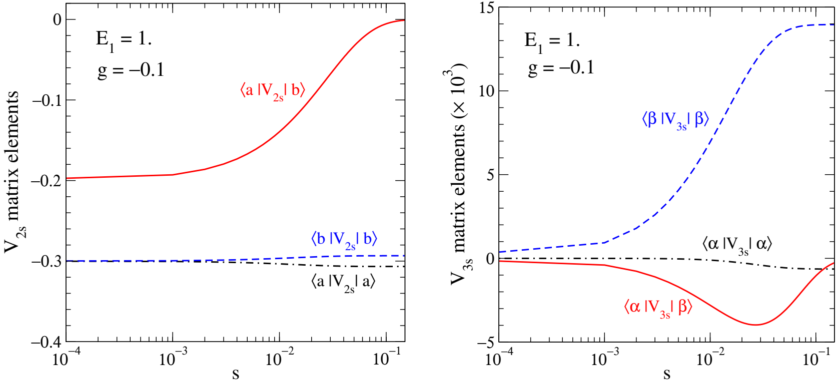
$-$ 

$-(\epsilon_a - \epsilon_b)^2 \, V_s^{(2)}(a,b)$ $\qquad$ $\sum_c [(\epsilon_a - \epsilon_c) - (\epsilon_c - \epsilon_b)] \, V_s^{(2)}(a,c) \, V_s^{(2)}(c,b)$

$\dfrac{dV_s^{(3)}}{ds} =$  $+$  $+$  $+$  $+ \cdots$

Simple Model: Flow for Weak Coupling

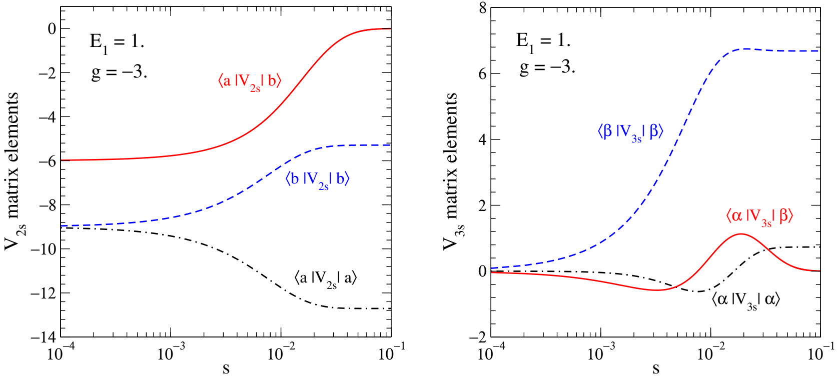
- Two-body potential on left; three-body on right



- See arXiv:0708.1502

Simple Model: Flow for Strong Coupling

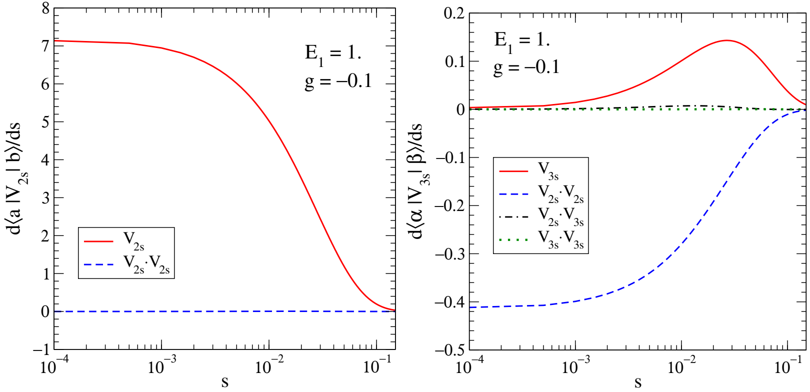
- Two-body potential on left; three-body on right



- See arXiv:0708.1502

Simple Model: Hierarchy of Terms

- Weak coupling: two-body V_{2s} on left; three-body V_{3s} on right
- Only off-diagonal terms here



- “Tree-level” diagrams dominate
- See arXiv:0708.1502

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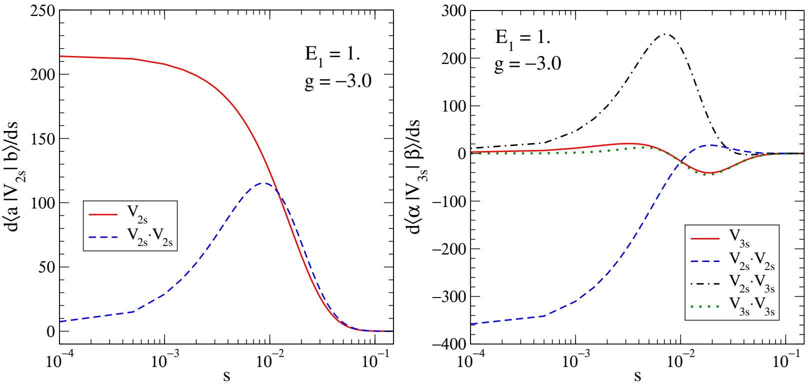

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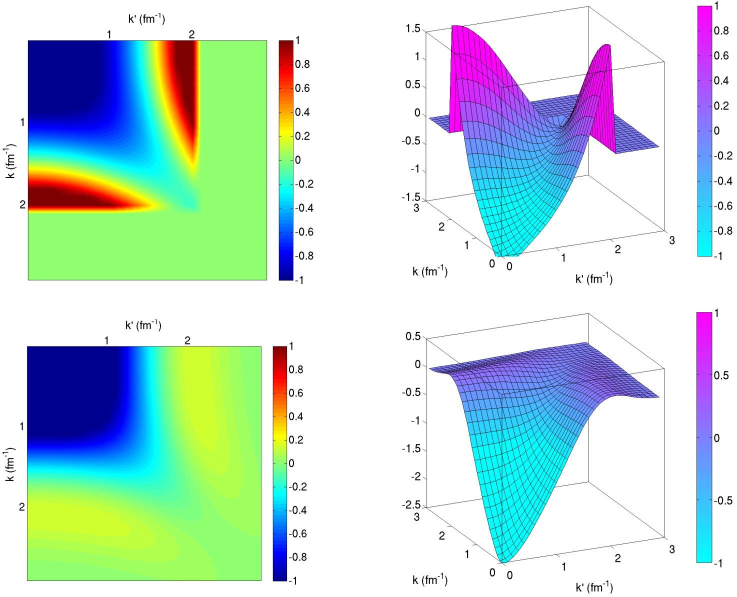
SRG for Few-Body: Summary and Outlook

- Well-defined unitary transformations for all operators
  - simple diagrammatic rules for many-body Hamiltonian
  - key question: are many-body forces under control?
- SRG-evolved two-body interactions like soft regulated $V_{\mathrm{low}\,k}$
  - same reduced correlations, improved convergence
  - but technically easier to evolve NNN, etc.
- Work in progress:
  - testing and documenting decoupling [E. Jurgenson]
  - operator evolution and factorization [E. Anderson]
  - T vs. H_D evolution [S. Glazek, R. Perry]
  - three-body evolution test cases [S. Bogner, rjf, R. Perry]
  - calculations with NCSM [P. Maris, J. Vary], coupled cluster [D. Dean, G. Hagen], initially with NNN fit to N^2LO form
  - other few-body bound-state and scattering approaches

Chiral N^3LOW vs. SRG Evolved: Potentials



Chiral N^3LOW vs. SRG Evolved: Convergence