

The Similarity Renormalization Group (SRG)

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On the web: <http://www.physics.ohio-state.edu/~ntg/srg/>

Motivation for SRG

- Want to use minimal degrees of freedom for precision nonperturbative microscopic low-energy calculations
- As resolution increases degrees of freedom increase without bound
- Need a formalism that rigorously controls resolution → **Exact Renormalization Group**
- The Similarity Renormalization Group is an Exact RG

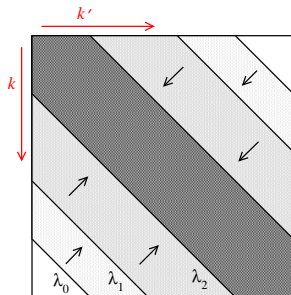
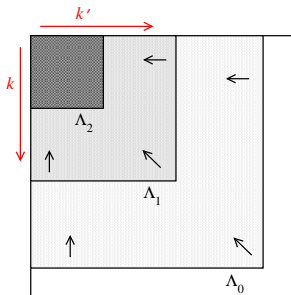
Motivation for SRG II

- The SRG produces band-diagonal interactions
 - Unitary transformation designed to decouple low- and high-energy states
 - All observables preserved
 - No relevant changes to low energy observables even when high momenta are removed
 - Natural hierarchy of many-body forces maintained

Motivation for SRG III

- Designed for light-front QCD
 - Requires Hamiltonian formulation and suitable renormalization
 - $P^- = \frac{P_{\text{perp}}^2 + m^2}{P^+}$
 - Wee partons have high energy \rightarrow vacuum decouples
 - Constituent picture emerges naturally
- In QED:
 - $\lambda^2 < m^2 \rightarrow$ electron-positron pairs decouple
 - $\lambda^2 < \alpha m^2 \rightarrow$ photons decouple

The Similarity Renormalization Group (SRG)



The Similarity Renormalization Group (SRG)

$$H_s = U(s) H U^\dagger(s) \equiv T + V_s$$

$$\frac{dH_s}{ds} = [\eta(s), H_s] \quad \text{where} \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

$$\eta(s) = [G_s, H_s] \quad \implies \quad \frac{dH_s}{ds} = [[G_s, H_s], H_s]$$

Projected onto partial-wave momentum representation
($G_s = T$):

$$\begin{aligned} \frac{dV_s(k, k')}{ds} &= -(\epsilon_k - \epsilon_{k'})^2 V_s(k, k') \\ &+ \frac{2}{\pi} \int_0^\infty q^2 dq (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_s(k, q) V_s(q, k') \end{aligned}$$

“Simple” example: delta function potential

$$H_\Lambda = P^2 - \alpha_\Lambda \Lambda^{2-d} \delta_\Lambda^d(X)$$

$$H_\Lambda |\psi_\Lambda\rangle = E |\psi_\Lambda\rangle$$

$$\begin{aligned} \langle k | H | \psi \rangle &= k^2 \psi(k) - \alpha \Lambda^{2-d} \int d^d q \langle k | \delta_\Lambda | q \rangle \psi(q) \\ &= E \psi(k) \end{aligned}$$

- Choice of initial regulator affects no qualitative results

$$\langle k | \delta_\Lambda | q \rangle = \left(\frac{1}{2\pi}\right)^d \theta(\Lambda - |k|) \theta(\Lambda - |q|)$$

- Focus on s-wave:

$$k^2 \psi(k) - \frac{\alpha \Lambda^{2-d} \Omega_d}{(2\pi)^d} \int_0^\Lambda dq q^{d-1} \psi(q) = E \psi(q)$$

Introduce discretization, exponentially spaced grid

$$q = q_0 e^x \rightarrow q_0 e^{an}$$

- Discretize x with uniform spacing
- Exponentially small discretization error and cutoff 'error'

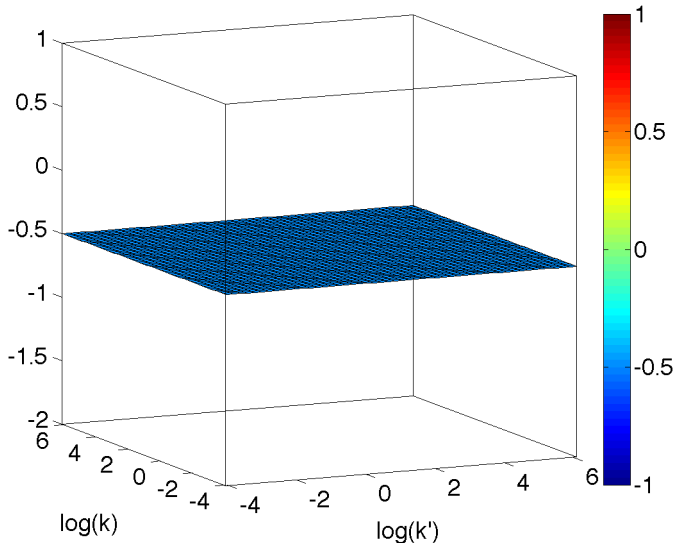
$$q_n = b^n$$

$$\begin{aligned} \frac{dV_{mn}}{ds} = & -(b^{2m} - b^{2n})^2 V_{mn} \\ & + c_d \sum_k b^{dk} (b^{2m} + b^{2n} - 2b^{2k}) V_{mk} V_{kn} \end{aligned}$$

- We want $b \rightarrow 1$ limit, but $b \rightarrow \infty$ limit can be solved analytically and yields main features of numerical results.

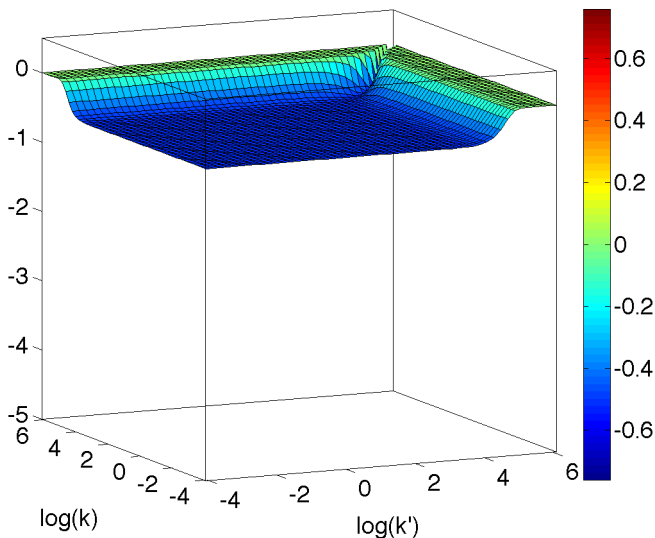
$$G = [T, H], g = 0.5, E_{bd} = 2.4674$$

$g_0=0.50$ $b=1.2247$ $M=-20$ $N=30$ $\lambda = \infty$



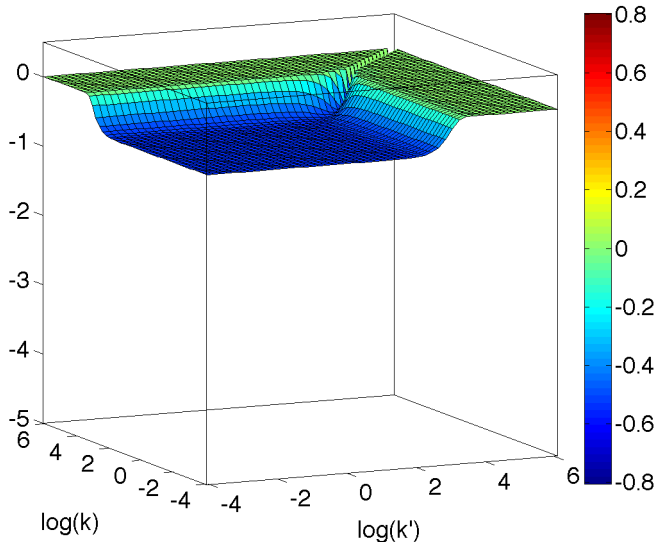
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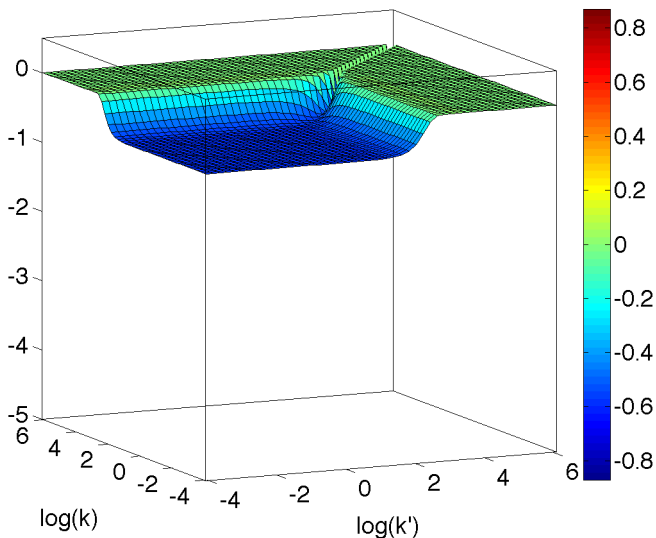
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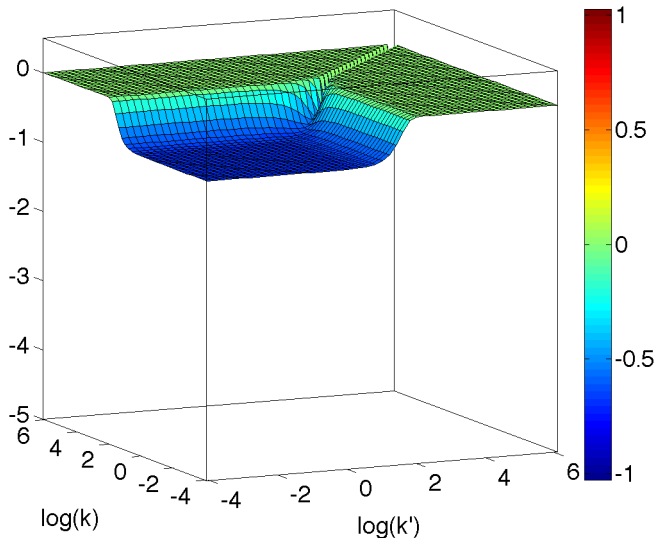
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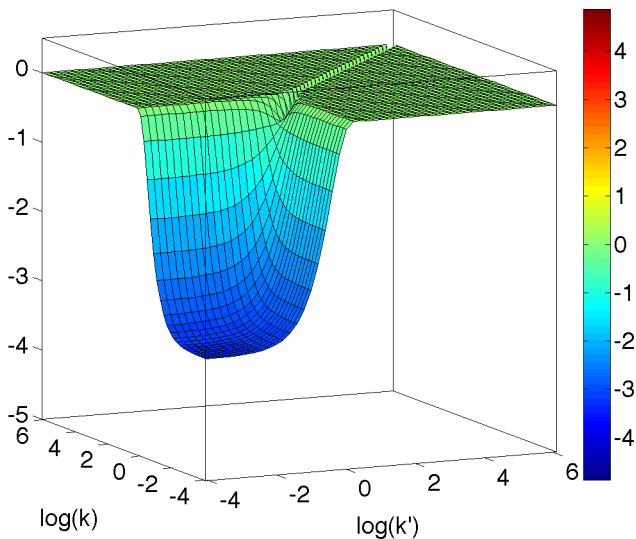
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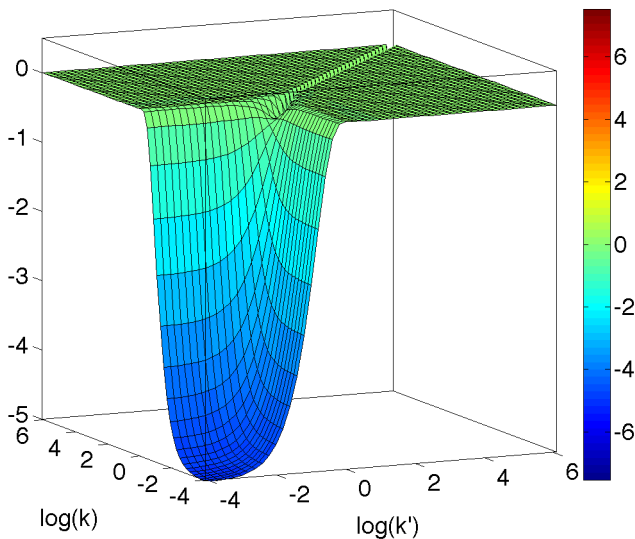
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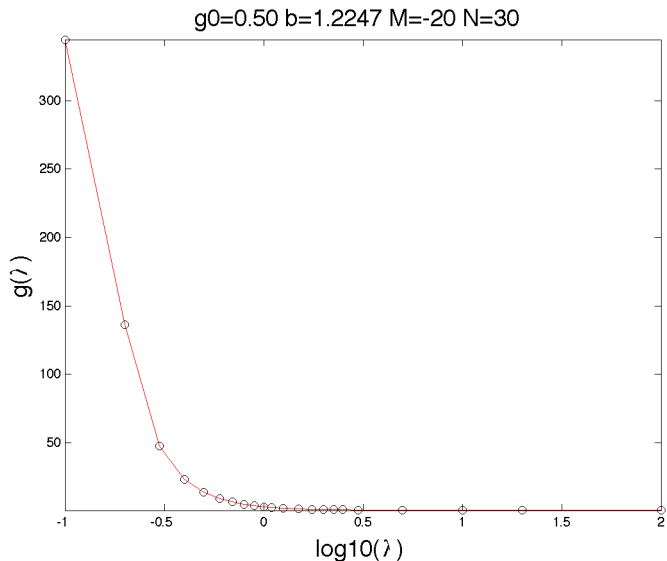


$$G = [T, H], g = 0.5, E_{bd} = 2.4674$$

$g_0=0.50$ $b=1.2247$ $M=-20$ $N=30$ $\lambda=0.80$

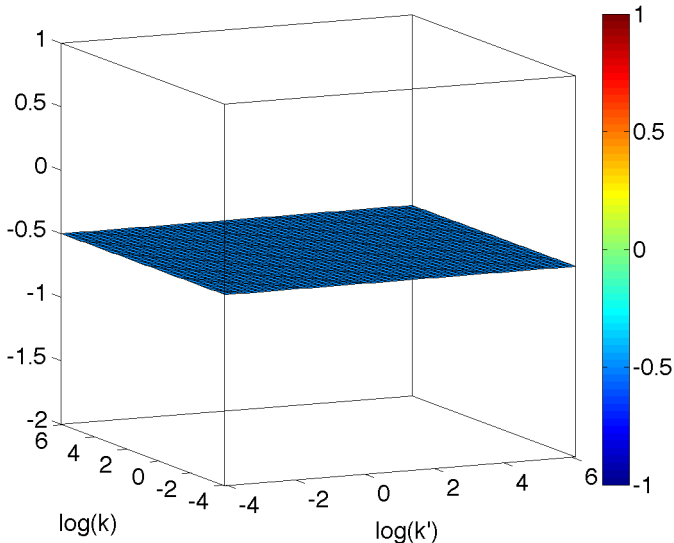


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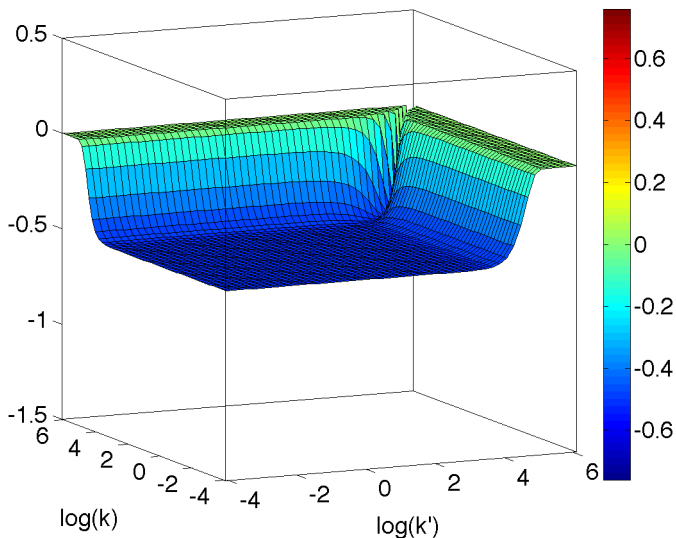
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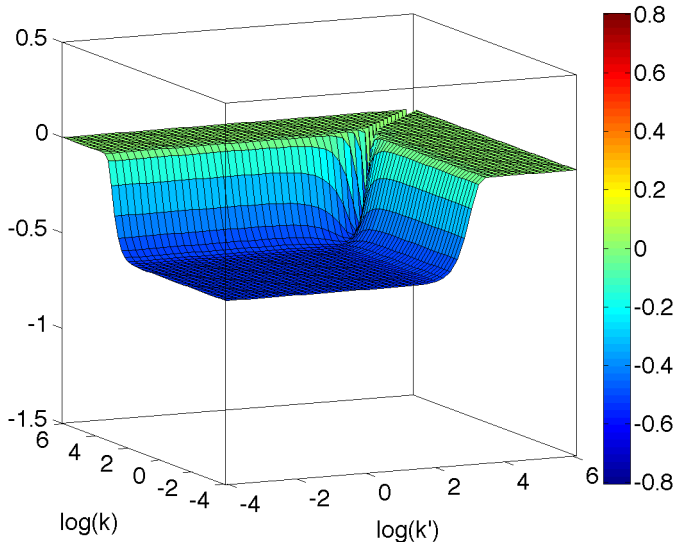
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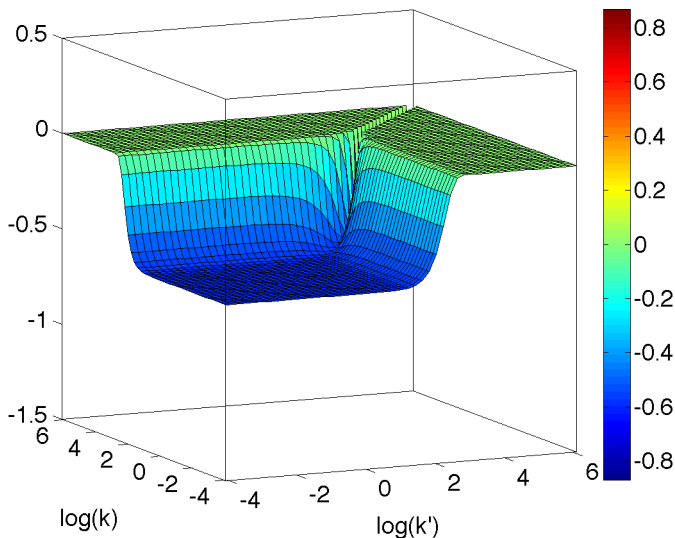
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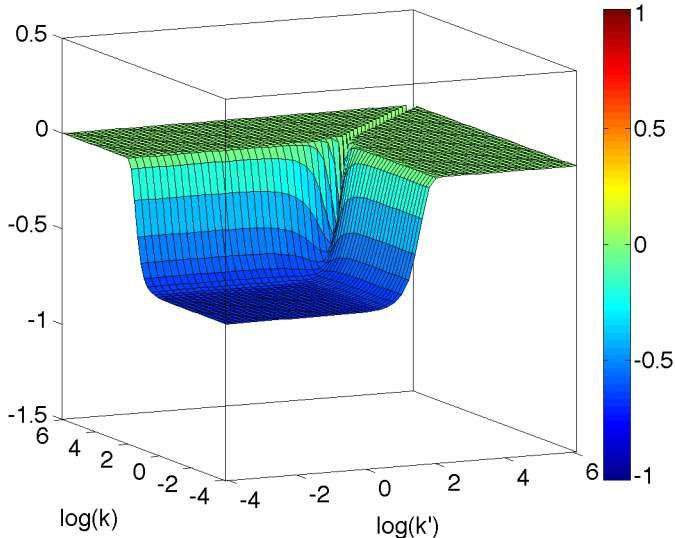
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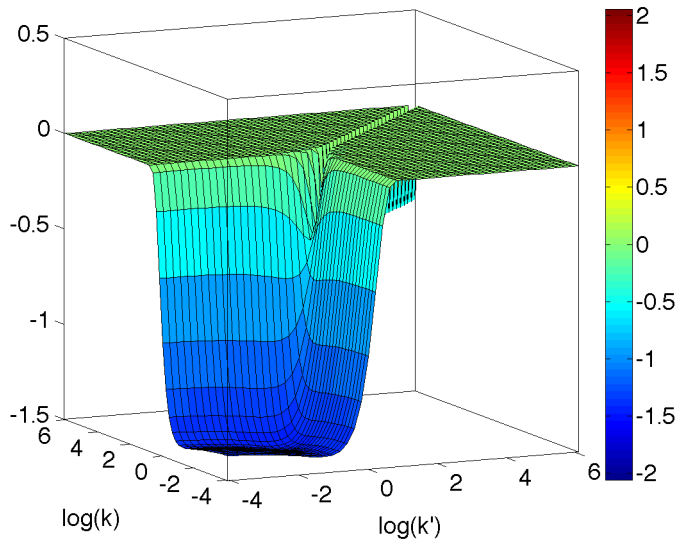
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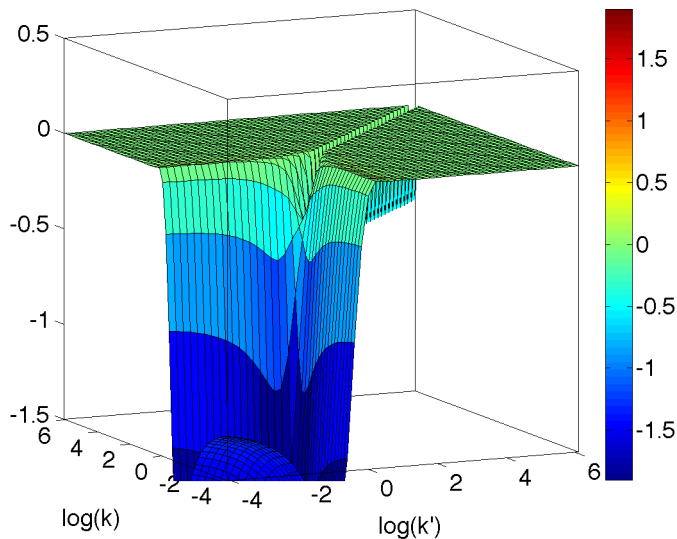
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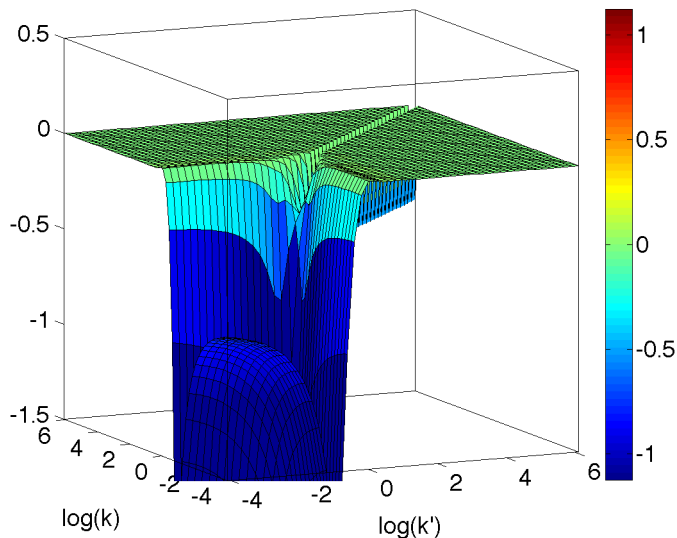
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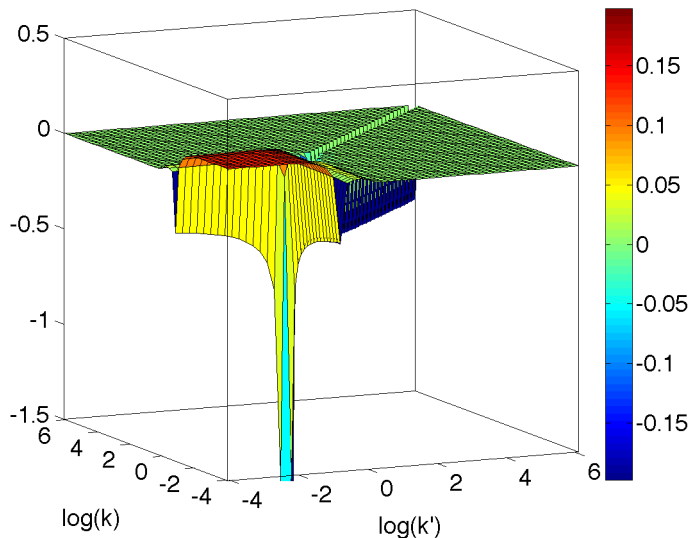
$$G = [H_{diag}, H], g = 0.5, E_{bd} = 2.4674$$

$g_0=0.50$ $b=1.2247$ $M=-20$ $N=30$ $\lambda=0.90$



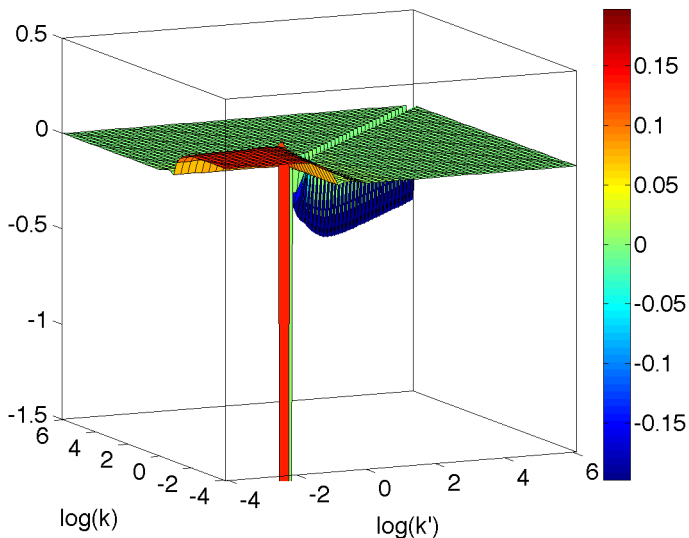
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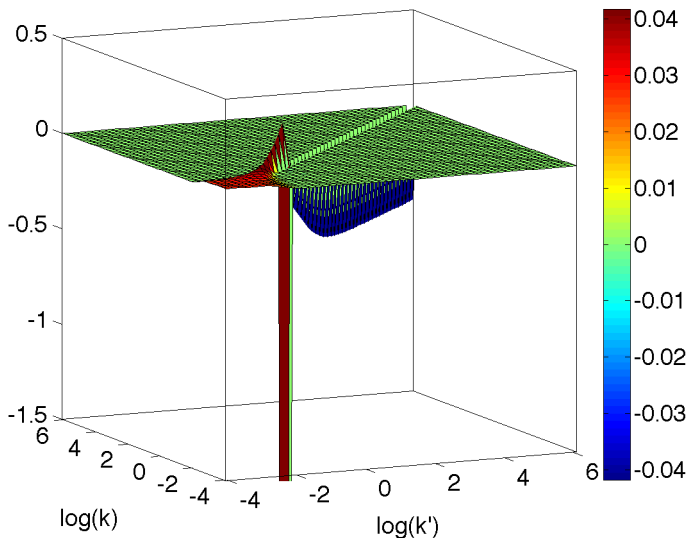
$$G = [H_{diag}, H], g = 0.5, E_{bd} = 2.4674$$

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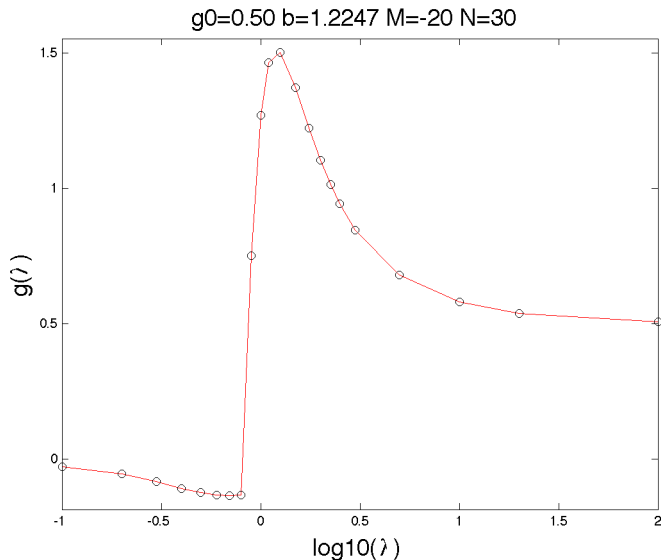


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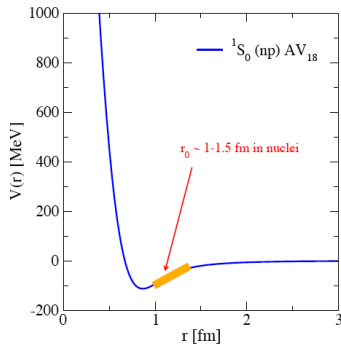


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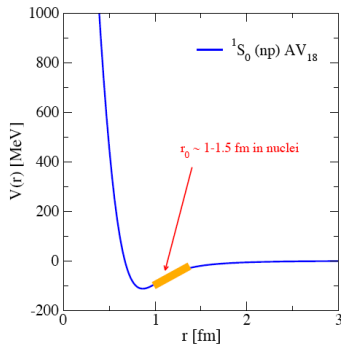
Sources of Nonperturbative Physics for NN

- 1 Strong short-range repulsion (“hard core”)
- 2 Iterated tensor interaction
- 3 Near zero-energy bound states



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 - Diagrammatic analyses become hopelessly complicated.



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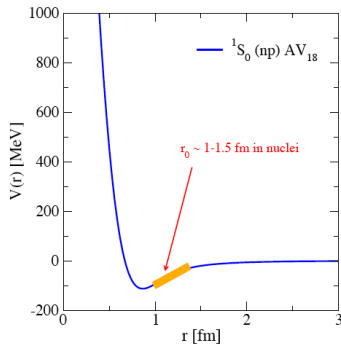
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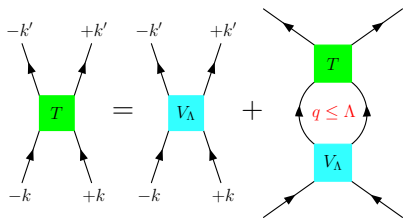
- However ...

- the potential depends on the resolution \implies **cutoff dependent**
- We can use a RG to control the cutoff.

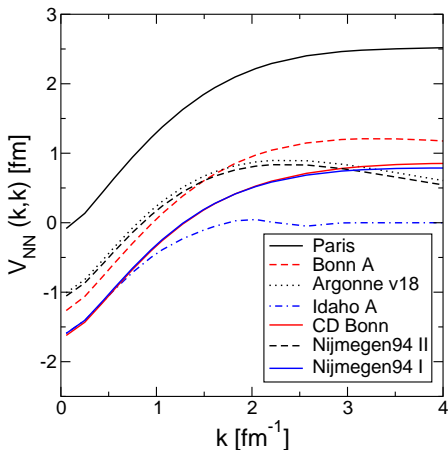


Gameplan: Exploit Variable Cutoff Potential

Bogner, Kuo, Schwenk

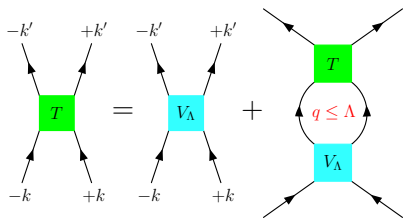


- Require $\frac{dT}{d\Lambda} = 0$
 \implies renormalization group equation for V_Λ
- Run from $\Lambda_B = 25 \text{ fm}^{-1}$ to $\Lambda = 2 \text{ fm}^{-1} \sim E_{\text{lab}} \doteq 350 \text{ MeV}$

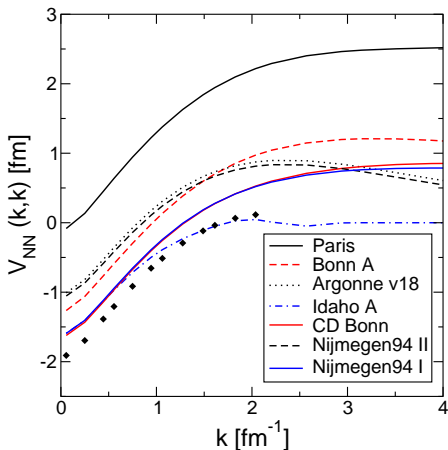


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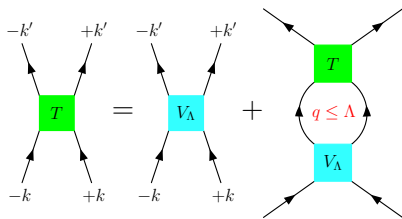


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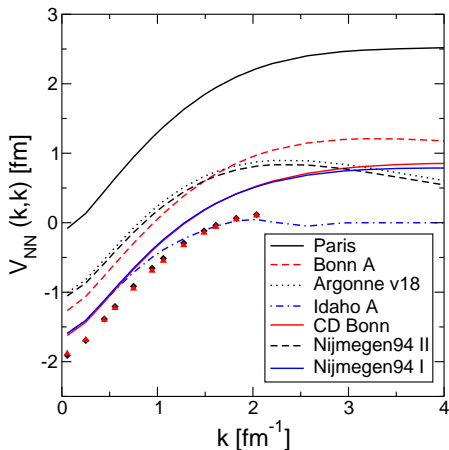


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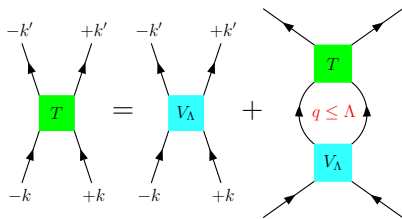


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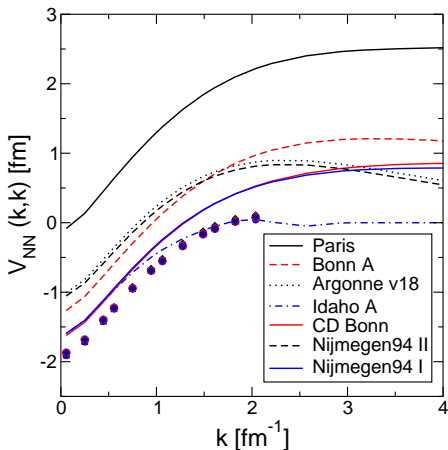


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- Same long distance physics
 \implies collapse to “ $V_{\text{low } k}$ ”!



The Similarity Renormalization Group (SRG)

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$$\frac{dH_s}{ds} = [\eta(s), H_s] \quad \text{where} \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

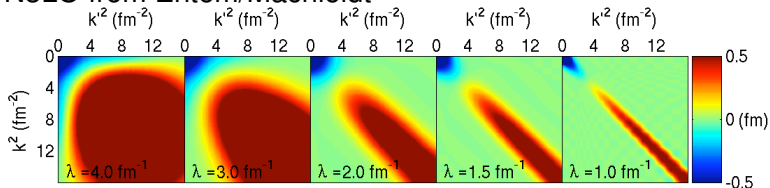
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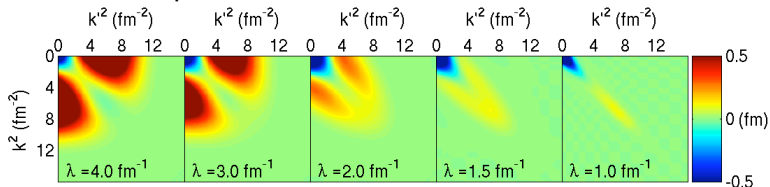
Typical comparison in 1S_0

- N3LO from Entem/Machleidt



1S_0 potential #12

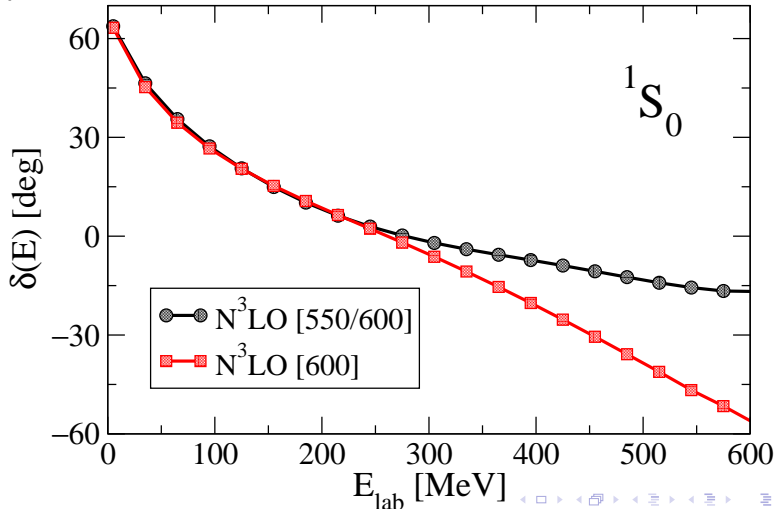
- N3LO from Epelbaum



1S_0 potential #32

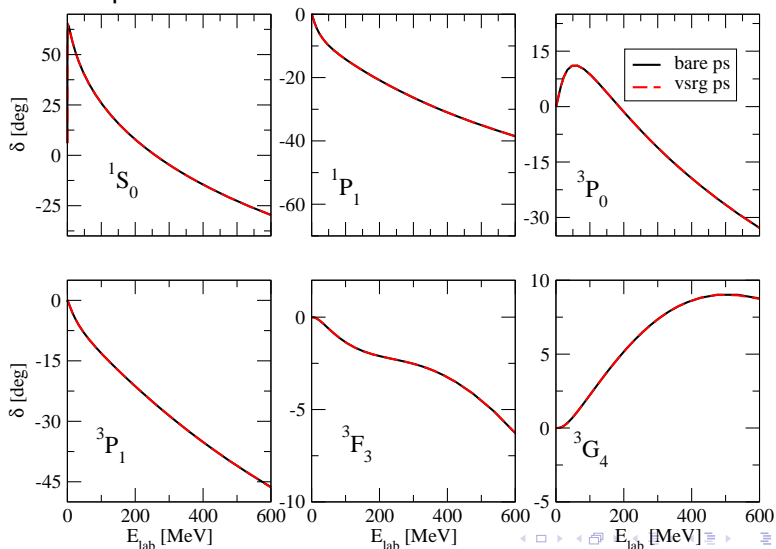
Phase shifts do not change

- S-wave phase shifts from the two chiral EFT $N^3\text{LO}$ potentials.



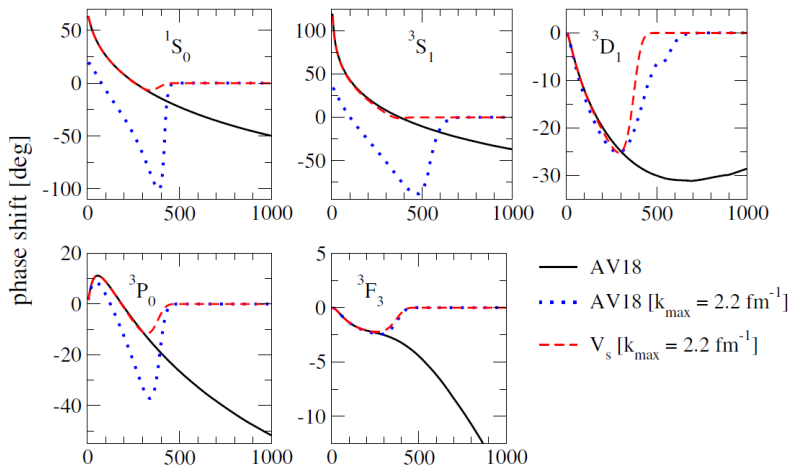
Bare vs. SRG phase shifts

- Several partial waves



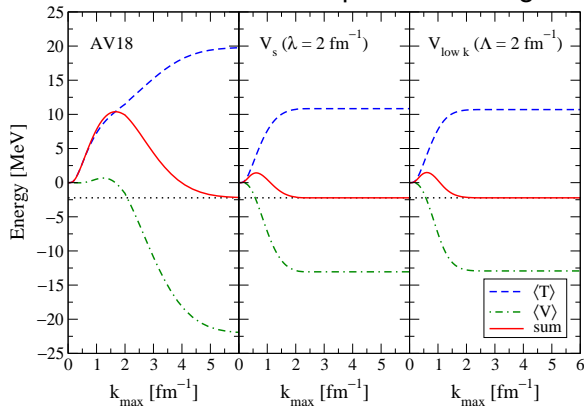
SRG decouples high momenta from low-energy observables

- Phase shifts with $V_s(k, k') = 0$ for $k, k' > k_{max}$



The strength of observables shifts considerably

- Distribution of kinetic and potential energies in the deuteron



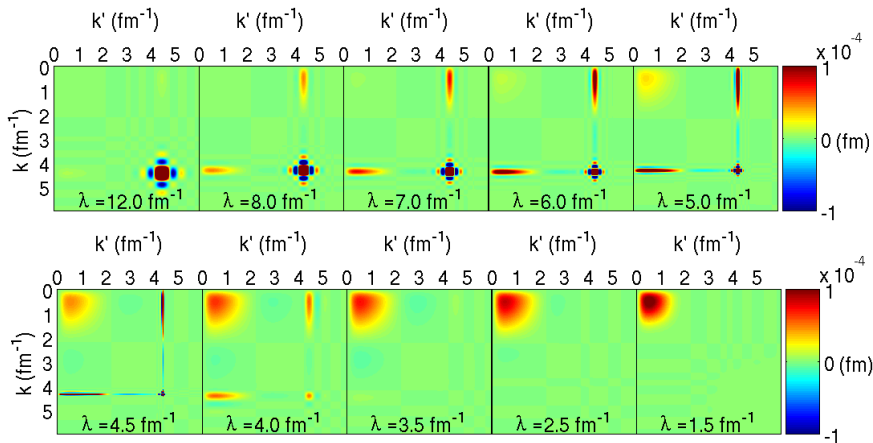
Every Observable Flows

- The evolution with s of any other operator O is given by the same unitary transformation, $O_s = U(s)OU^\dagger(s)$, which means that O_s evolves according to

$$\frac{dO_s}{ds} = [\eta(s), O_s] = [[T_{\text{rel}}, V_s], O_s].$$

- Consider the evolution of $\langle \psi_d | a^\dagger(q) a(q) | \psi_d \rangle$.
- The integral does not change, but the strength flows.
- Look at $\langle \psi_d(s) | k \rangle \langle k | U_s | q \rangle \langle q | U^\dagger | k' \rangle \langle k' | \psi_d \rangle$.

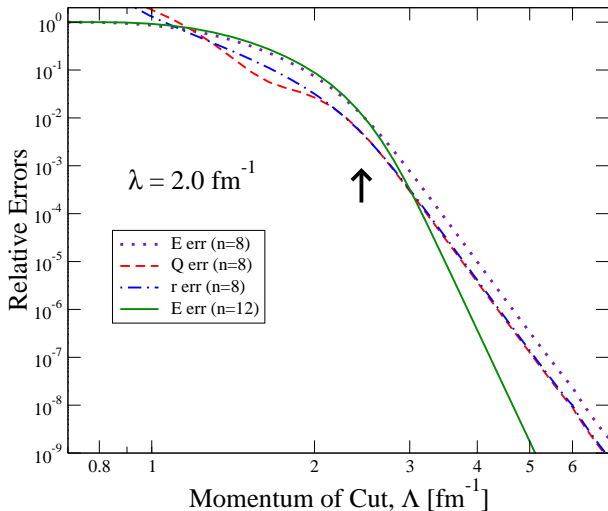
Flow of strength from $a^\dagger a$



Deuteron Observables

Deuteron Observables

- Binding Energy
- Quadrupole Moment
- RMS radius



Soft Potentials in History

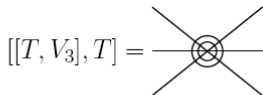
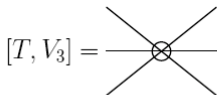
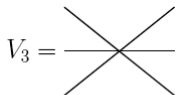
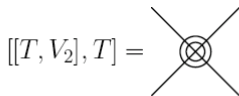
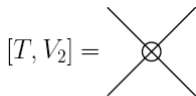
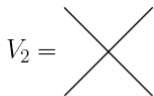
- There were active attempts to transform away hard cores and soften the tensor interaction in the late sixties and early seventies.
- But the requiem for soft potentials was given by Bethe:
“Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required.”

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- But the requiem for soft potentials was given by Bethe:
“Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required.”
- But that story is not complete: three-nucleon forces (3NF)!

Three-Body Interactions in the SRG


- Same SRG equation: $\frac{dH}{ds} = [[T, H], H]$
- $H = T + V_2 + V_3 + \dots$
- $V_2 = \frac{1}{2!} a_i^\dagger a_j^\dagger (ij|V_2|kl) a_l a_k$
- $V_3 = \frac{1}{3!} a_i^\dagger a_j^\dagger a_k^\dagger (ijk|V_3|lmn) a_n a_m a_l$




Fock space SRG equation

$$\bullet \frac{dV_2(s)}{ds} + \frac{dV_2(s)}{ds} + \dots =$$

$$[[T, V_2(s) + V_3(s)], T] + [[T, V_2(s) + V_3(s)], V_2(s) + V_3(s)] + \dots$$

$$\frac{dV_2}{ds} =$$


The diagram shows the derivative of the two-body potential with respect to the similarity parameter. It consists of three terms: a self-energy diagram (a circle with an 'X' inside), a two-body exchange diagram (two lines forming a loop with an 'X' on the left), and a two-body exchange diagram (two lines forming a loop with an 'X' on the right). The first and second terms are added, and the third term is subtracted.

$$\frac{dV_3}{ds} =$$


The diagram shows the derivative of the three-body potential with respect to the similarity parameter. It consists of four terms: a self-energy diagram (a circle with an 'X' inside), a two-body exchange diagram (two lines forming a loop with an 'X' on the left), a two-body exchange diagram (two lines forming a loop with an 'X' on the right), and a two-body exchange diagram (two lines forming a loop with an 'X' on the right). All four terms are added.

- 2-body and 3-body problems separate

Summary

- SRG provides a powerful new RG tool
- Produces well-defined RG-improved perturbation theory
- Provides new exact RG calculations
- SRG universality provides well-defined nuclear interactions
- SRG \rightarrow soft NN force & small compensating 3N force

- 1 Objective: Precise Strong Interaction Calculations at all Scales
- 2 Similarity Renormalization Group
- 3 Selecting the right degrees of freedom
- 4 3-Body
- 5 Summary