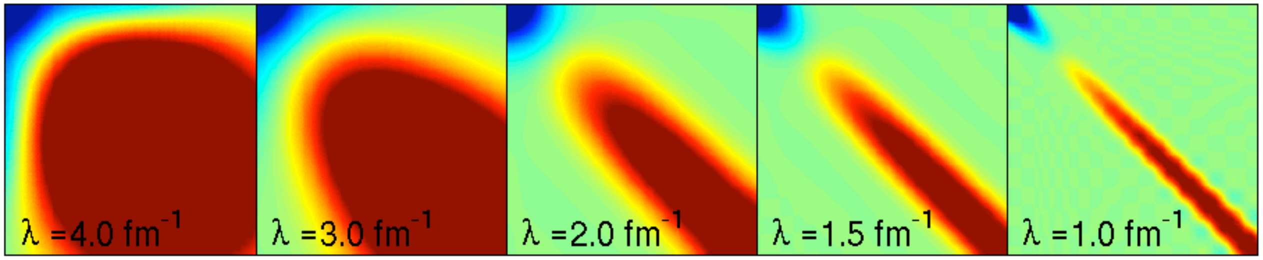


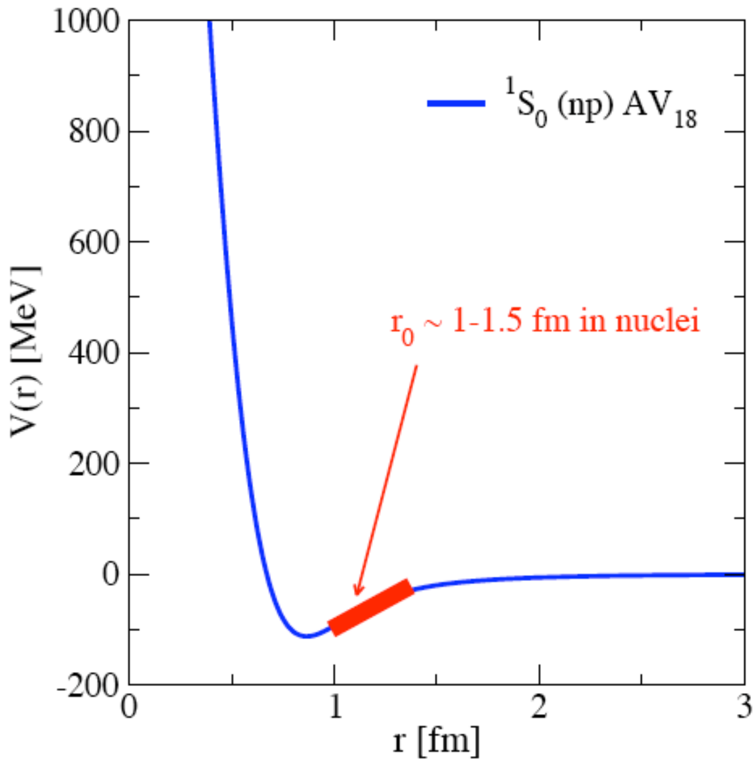
Low Momentum Interactions for Few- and Many-Body Systems

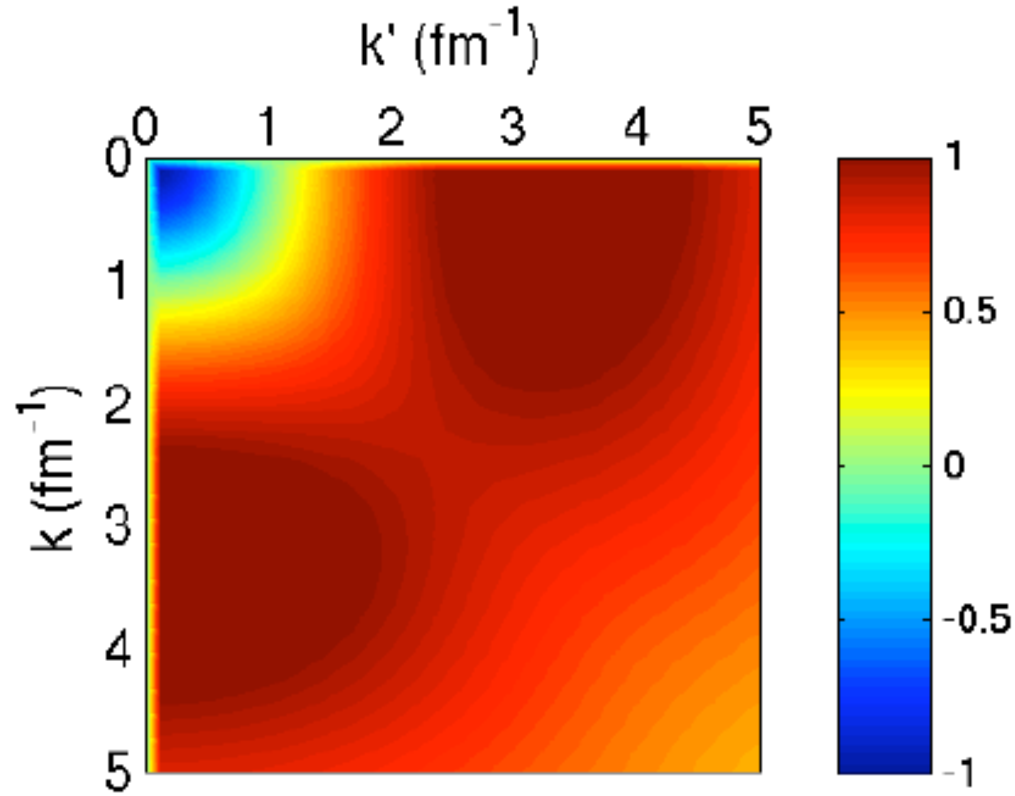
S.K. Bogner (NSCL/MSU)

NSCL

Collaborators: R.J. Furnstahl, S. Ramaman, R.J. Perry, A. Schwenk, A. Nogga, T. Duguet, V. Rotival, L. Platter

Non-Perturbative Features of NN Interactions





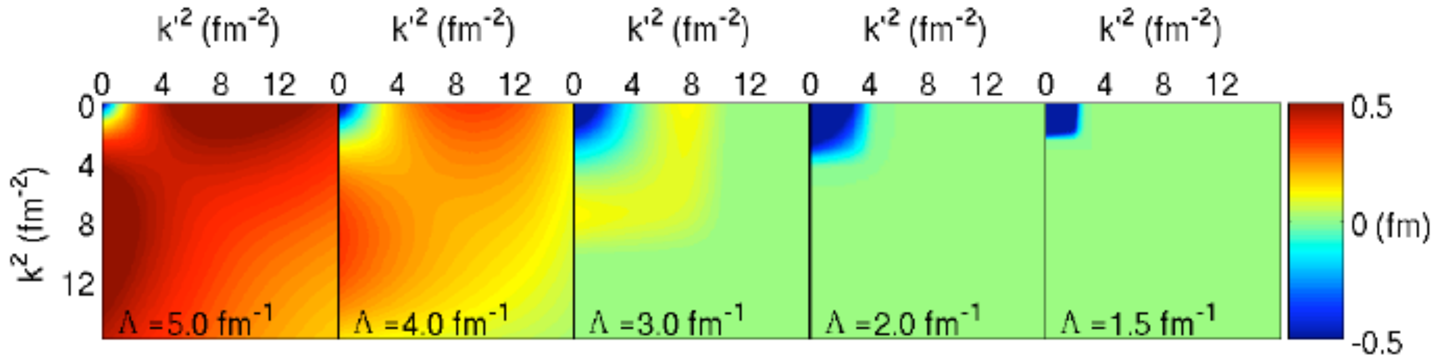
- strong core repulsion
- strong tensor forces
- deuteron pole
- pairing instability

} scheme-dependent, U.V. ***details*** (e.g., resolution scale Λ)

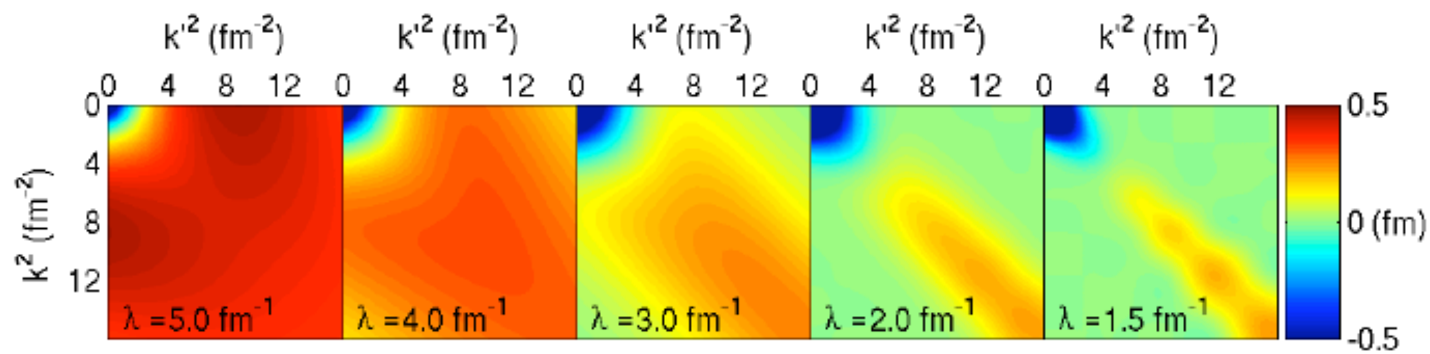
} scheme-**independent** (physical, independent of UV details)

2 Types of Renormalization Group Transformations

- "$V_{low\ k}$" lowers a cutoff Λ (sharp or smooth) in k, k'



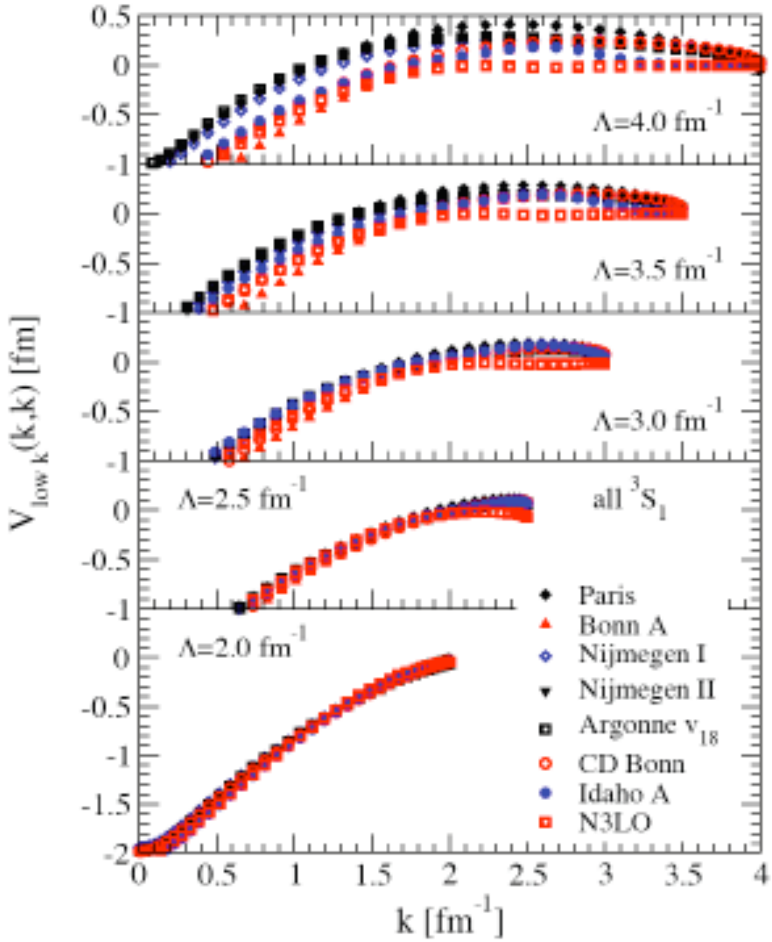
- Similarity RG drives H towards diagonal with flow parameter λ

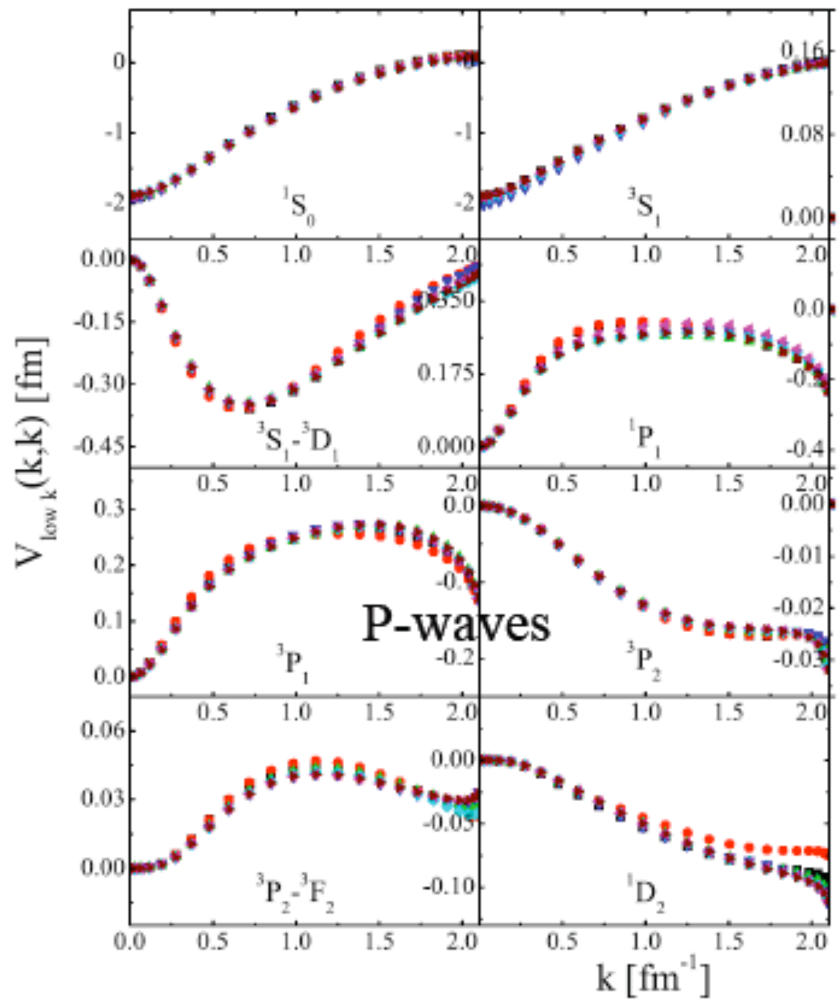


In both types of RG transformations:

- high and low momentum modes in Hamiltonian decoupled
- high quality description of NN scattering preserved

Low Momentum Universality of RG-evolved Interactions





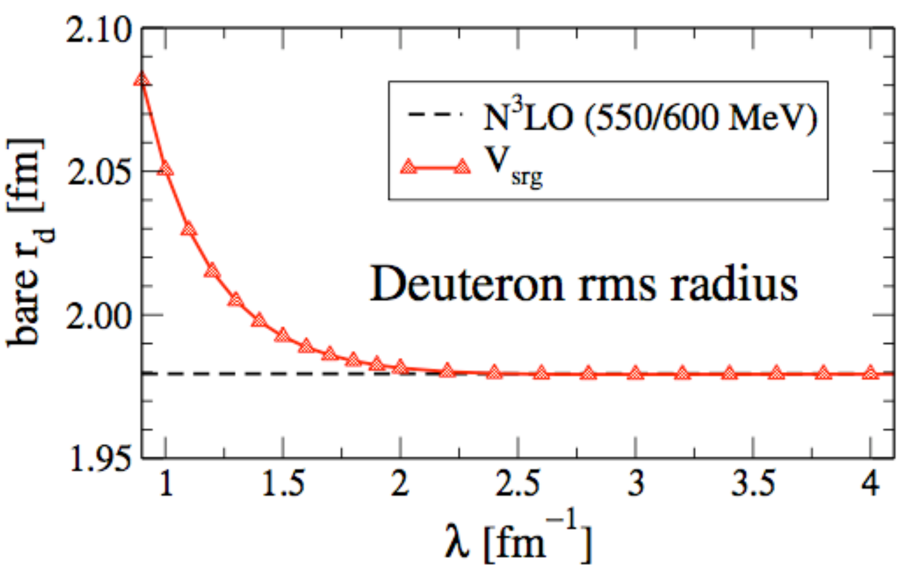
- **ALL** V_{NN} models flow to ≈ universal interaction as Λ lowered to the scale of the underlying NN scattering data

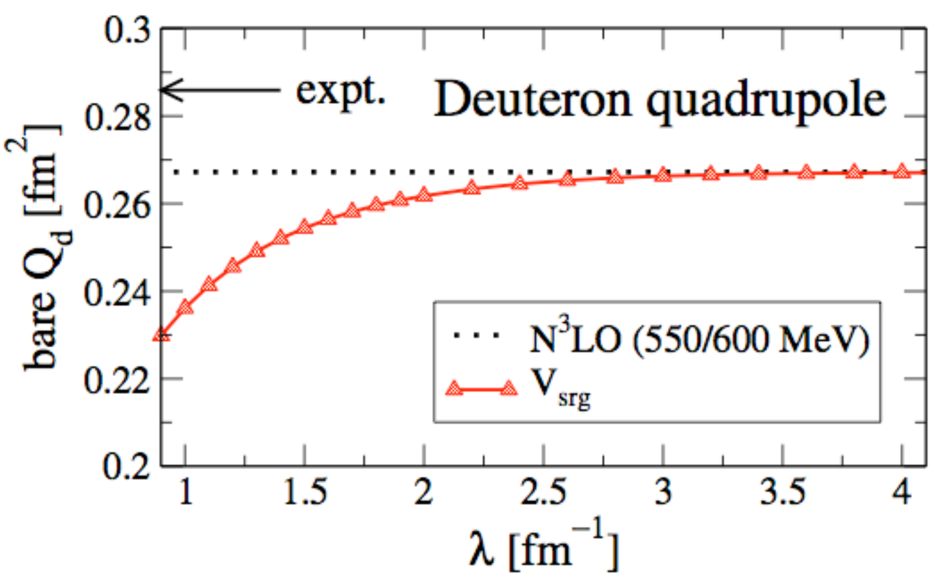
- RG evolution only affects short-distance structure

$$V_{\Lambda_0}(k,k') - V_\Lambda(k,k') = \widetilde{C}_0 + \widetilde{C}_2(k^2 + k'^2) + \cdots \quad \text{[nucl-th/0308036]}$$

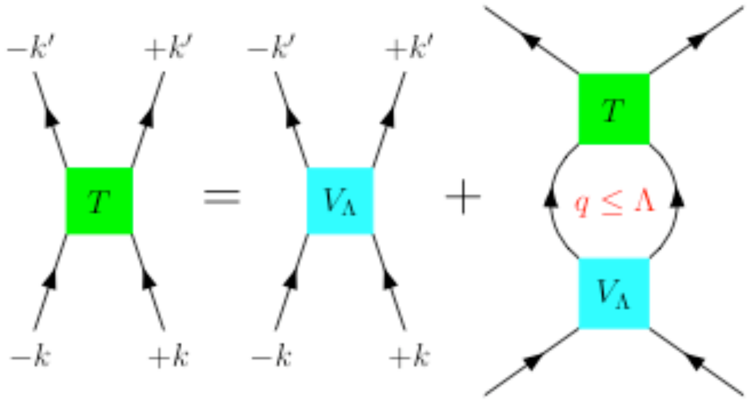
  - Long-range pion-physics (e.g., from χ-EFT) unchanged
  - higher order contact terms generated to maintain Λ-independent physics

- "Bare" long-range operators (Q_d, r_d) => weak Λ-dependence (in low energy states)





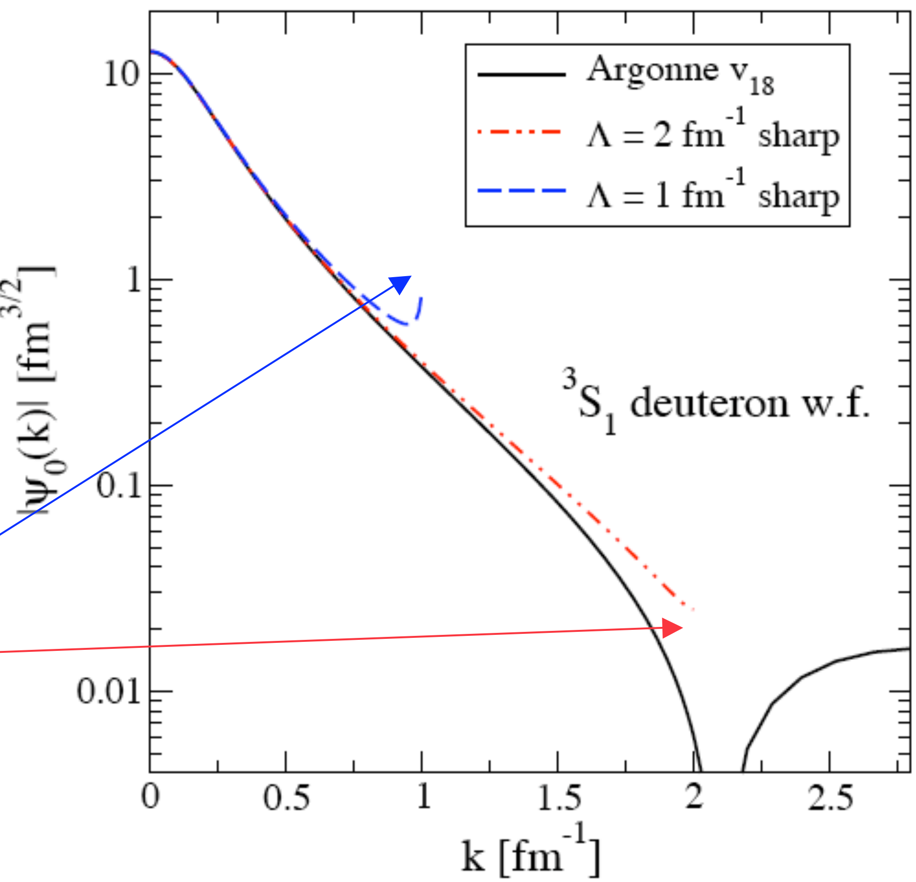
Technical Details -- Sharp versus Smooth Cutoffs



$$\frac{d}{d\Lambda}T = 0$$ ⇨ RGE for $V_{low\ k}$

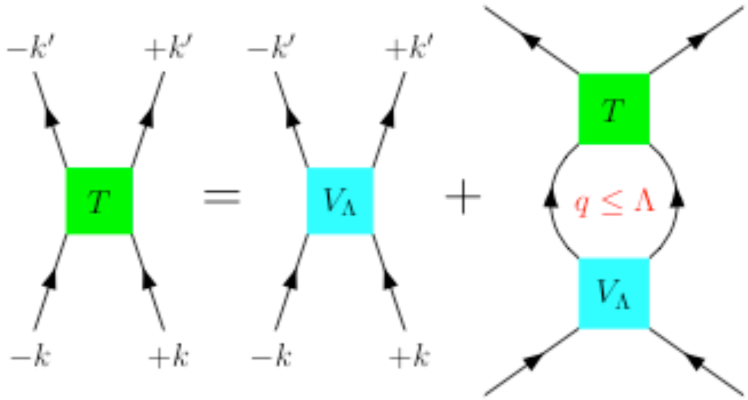
- Sharp cutoff $V_{low\ k}$ => $\theta(\Lambda - q)$
  - equivalent to Lee-Suzuki
  - deuteron w.f. discontinuous
  - r-space difficulties
  - tedious interpolations in FY eqns.
  - basis expansions slowly convergent

$V_{low\ k}$ Deuteron w.f. (sharp cutoff)



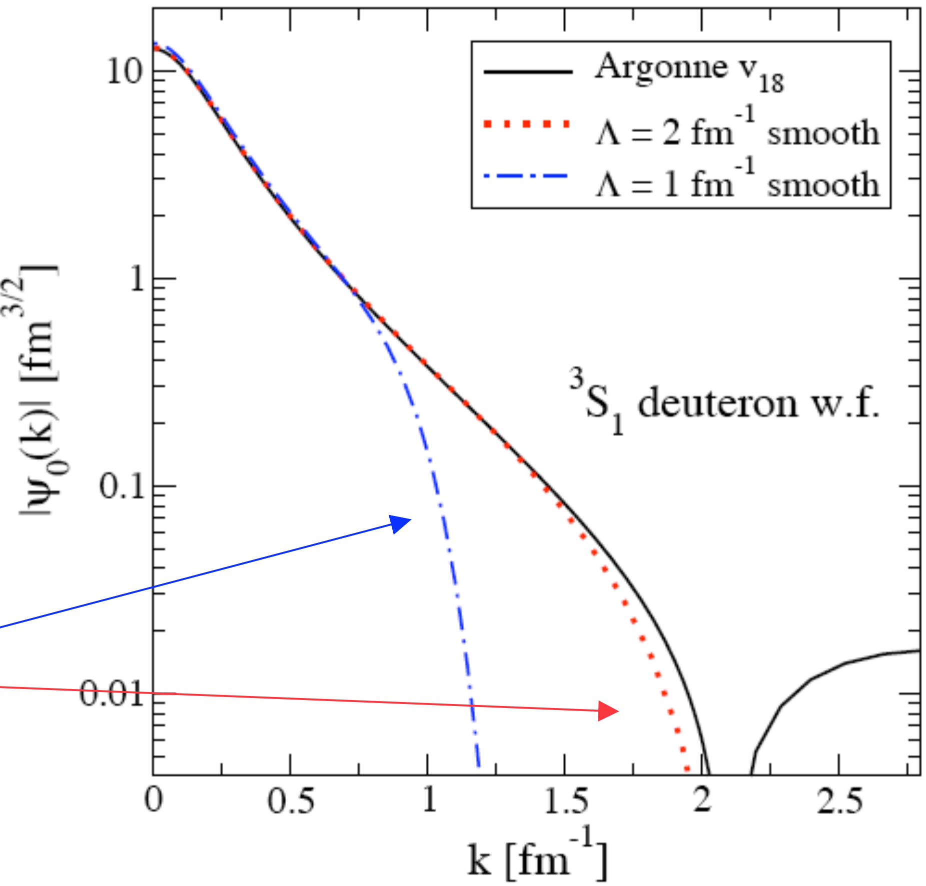
Must generalize $V_{low\ k}$ to smooth momentum cutoffs ($PQ \neq 0$)

Technical Details -- Sharp versus Smooth Cutoffs



$$\frac{d}{d\Lambda}T = 0 \Longrightarrow \text{RGE for } V_{\text{low k}}$$

$V_{\text{low k}}$ Deuteron w.f. (smooth)

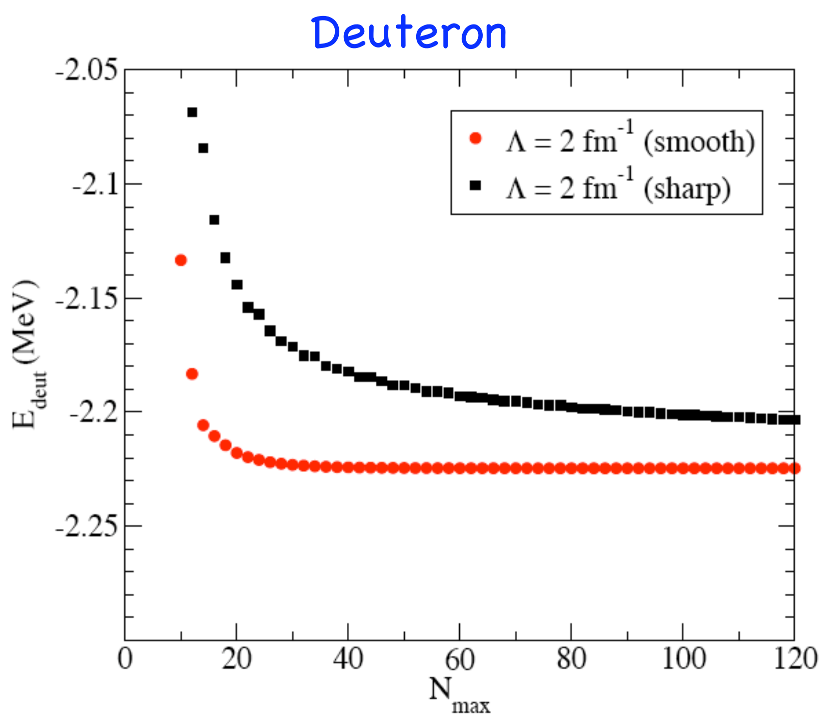


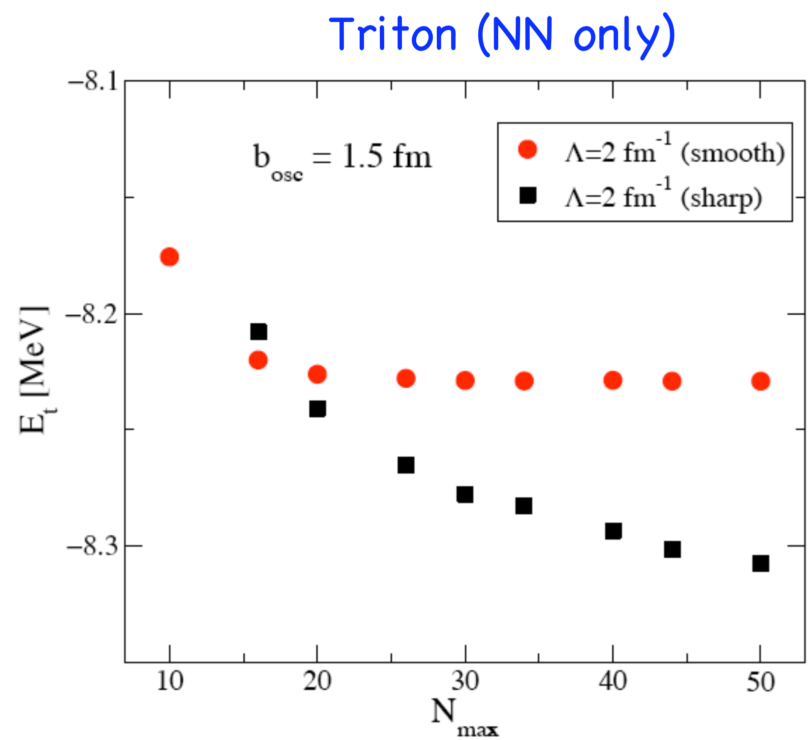
- $V_{\text{low k}}$ with a smooth cutoff
  - $\theta(\Lambda-q)$ --> $\exp[-(q/\Lambda)^{2n}]$
  - deuteron w.f. is continuous
  - r-space quantities well-defined
  - RG formulation generalizes Lee-Suzuki to PQ ≠ 0

(nucl-th/0602017)

NOTE: SRG interactions (see R.J. Furnstahl's talk) have the same features

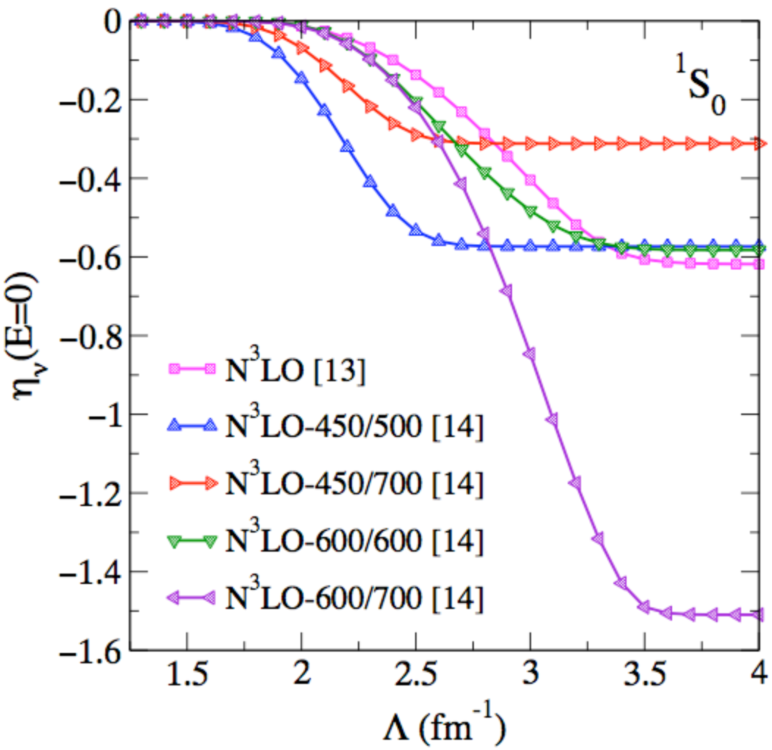
Convergence in a HO Basis Expansion (sharp vs. smooth $V_{low\ k}$)





- Smooth cutoff $V_{low\ k}$ gives rapid convergence in HO basis expansion
- Find similar results for similarity renormalization group interactions

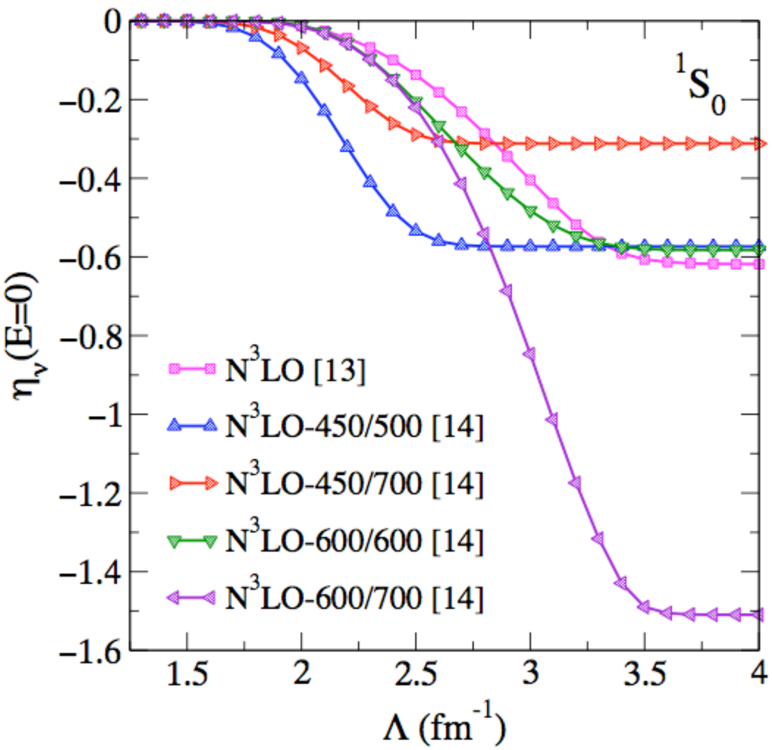
Mathematical conditions for "perturbativeness" via Weinberg eigenvalues



- Eigenvalues $\eta(E)$ of scattering kernel $G_0(E)V$
- perturbation theory works at E iff all $\eta(E) < 1$
- lowering Λ tames large η associated with UV (hard core, strong tensor)
- "physical" non-perturbativeness due to deuteron pole not changed by lowering Λ

HOWEVER...

Mathematical conditions for "perturbativeness" via Weinberg eigenvalues



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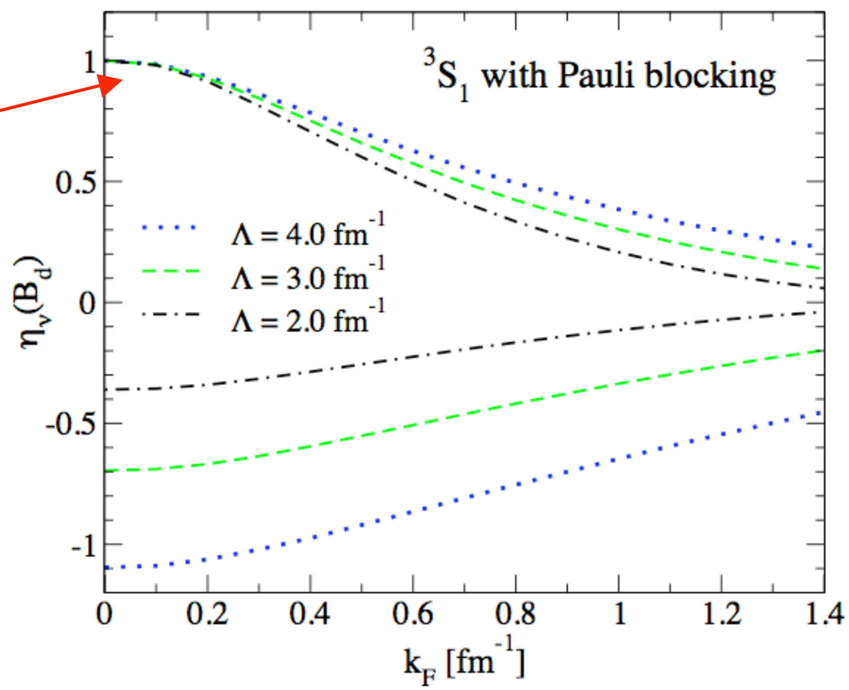
Pauli-blocking eliminates this non-perturbative behaviour at finite density



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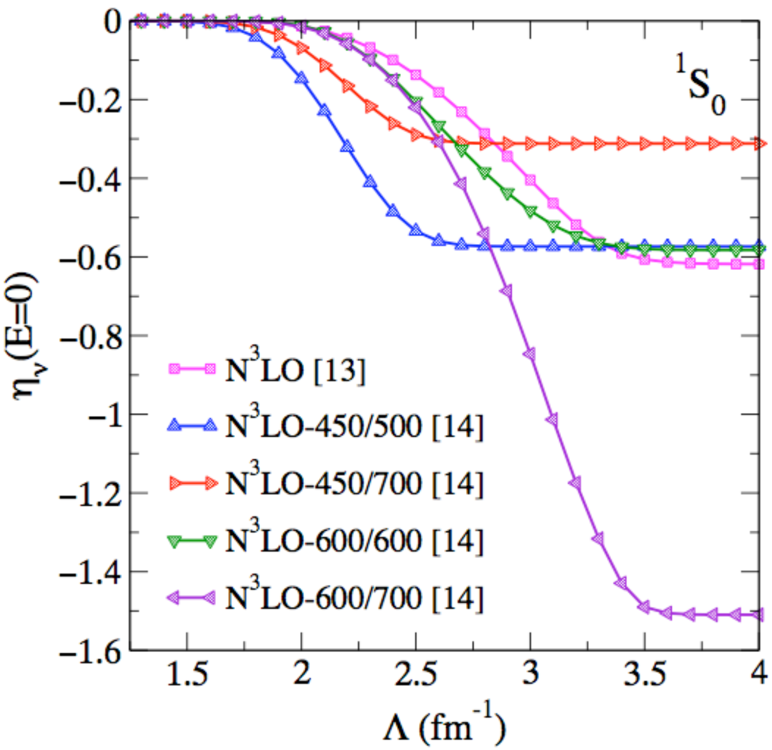

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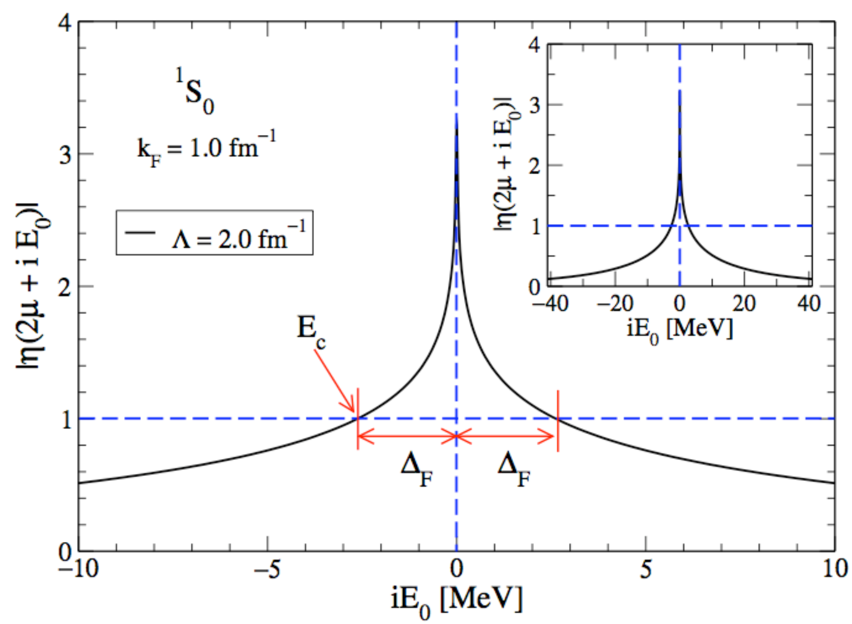
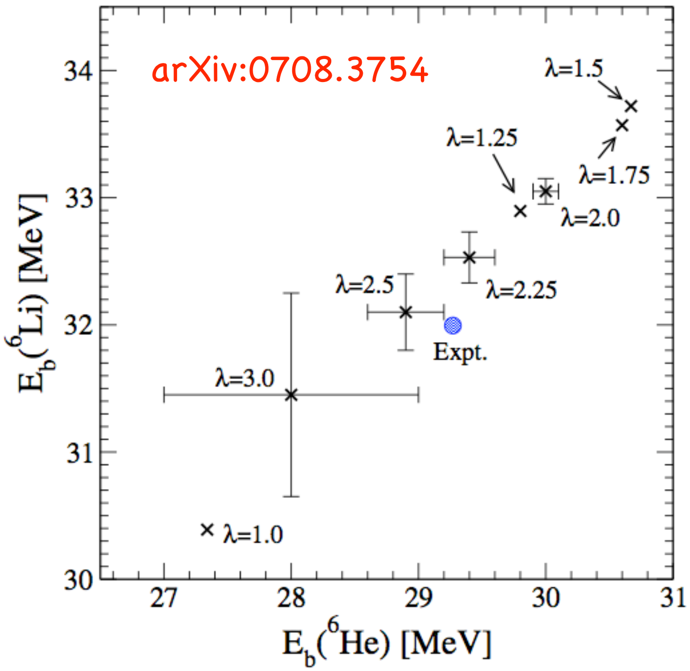
NOTE:

Pairing instability remains
Source of non-perturbative
physics for all Λ [arXiv:0709.0534]



Λ-dependence and 3N forces

^{6}Li-^{6}He Tjon-line



3N (and higher) forces arise from integrating out

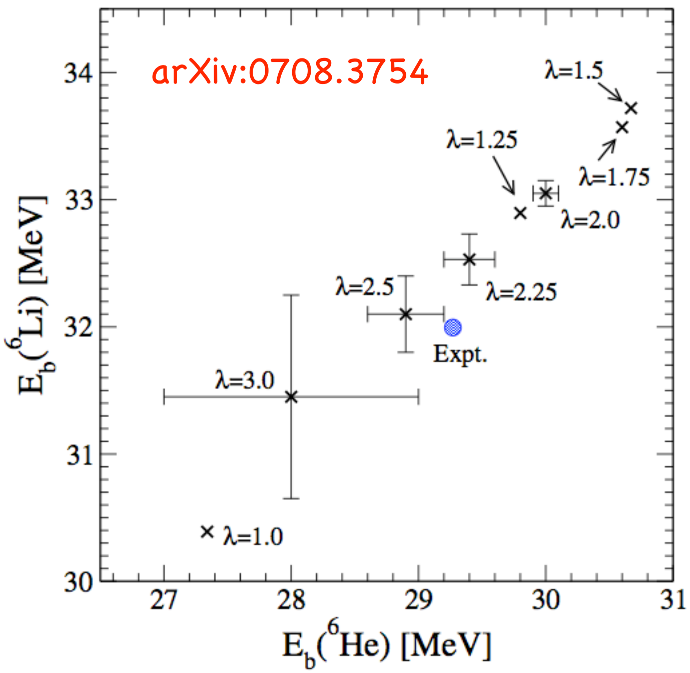
- excited states of nucleon (Δ, ...)
- relativistic effects
- high k modes

Λ-dependence => measure of what's missing

(3NF, higher order terms,...)

Λ-dependence and 3N forces

^{6}Li-^{6}He Tjon-line



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Λ-dependence => measure of what's missing

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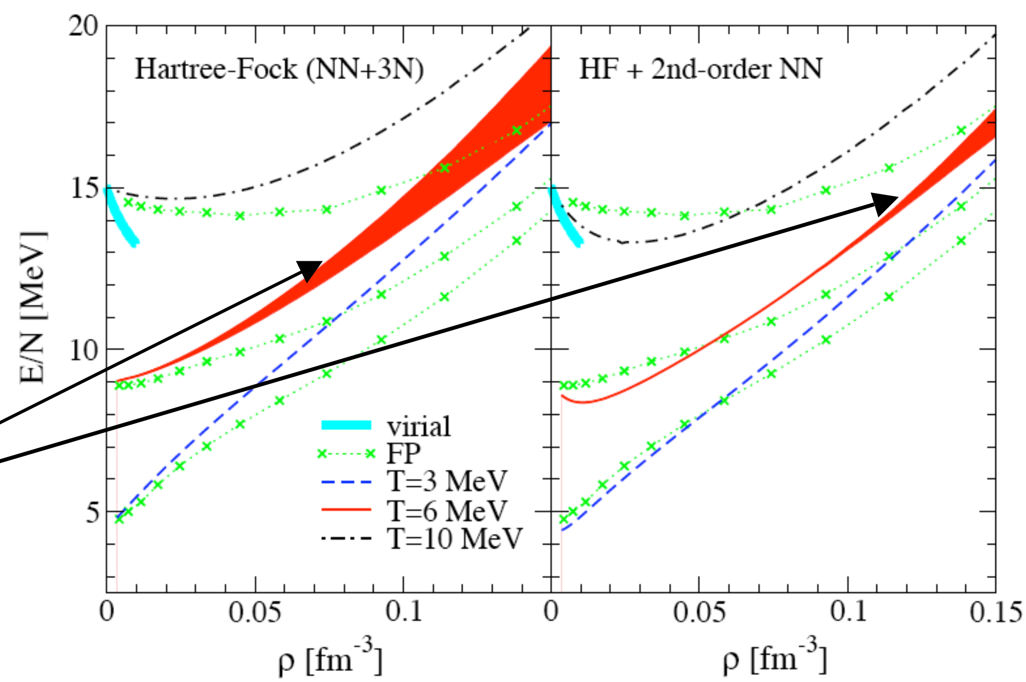
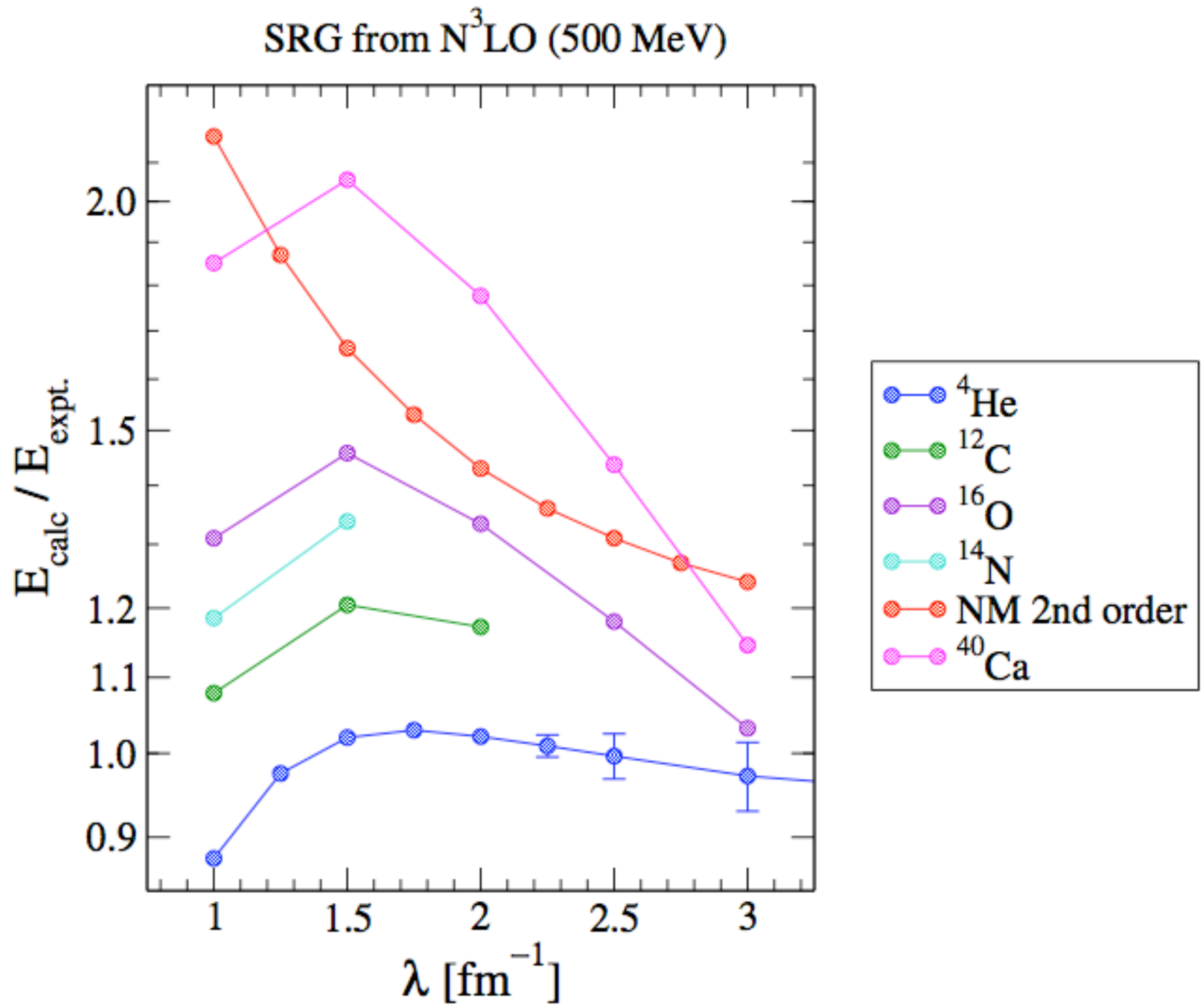
Finite T Neutron matter (Schwenk et.al.)



Theoretical error bands from Λ-dependence

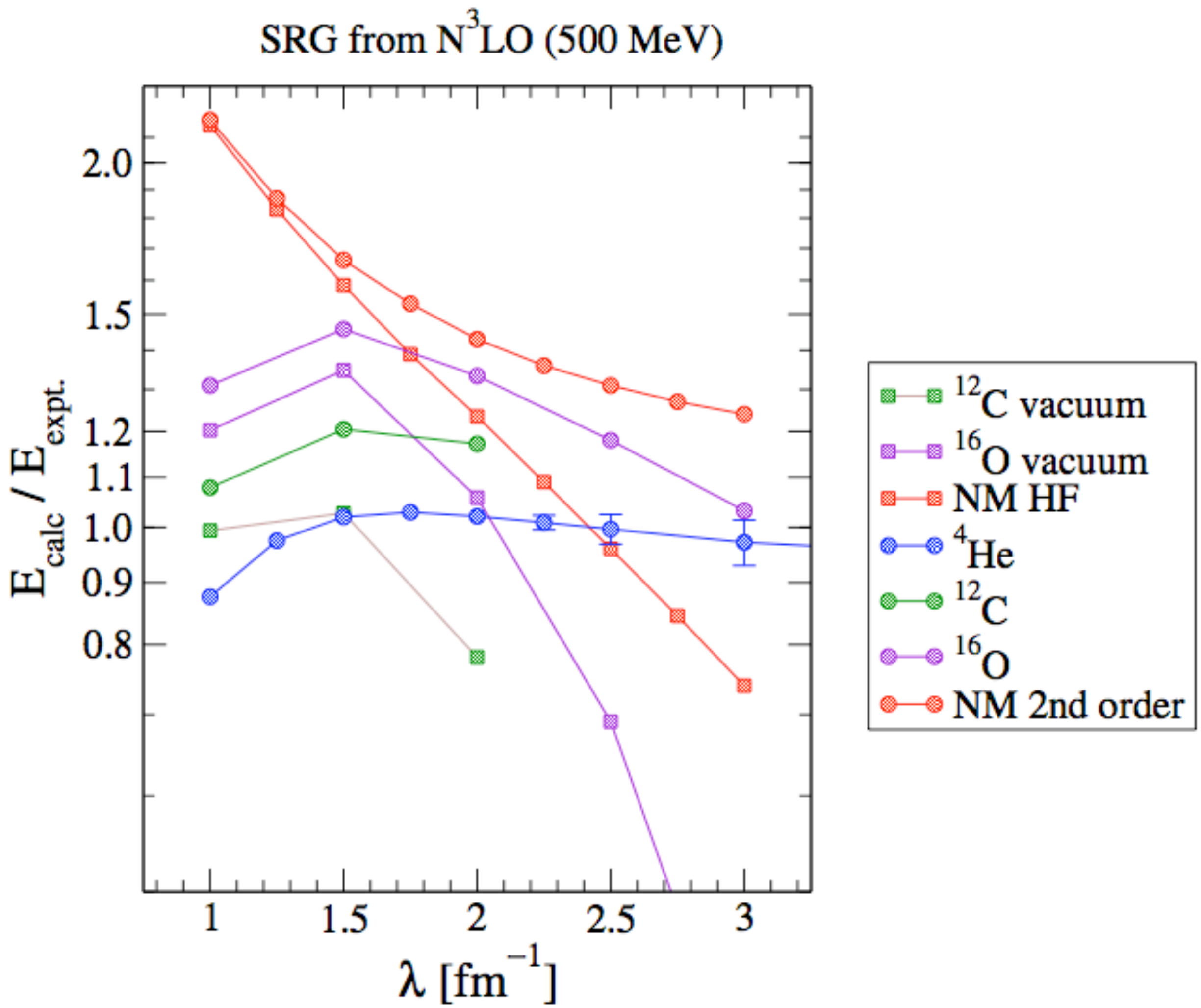
(4N forces, MB approximations,...)

Why you should not fine-tune Λ to minimize NNN



- Induced NNN (and 4N) grows w/density...BUT

Why you should not fine-tune Λ to minimize NNN

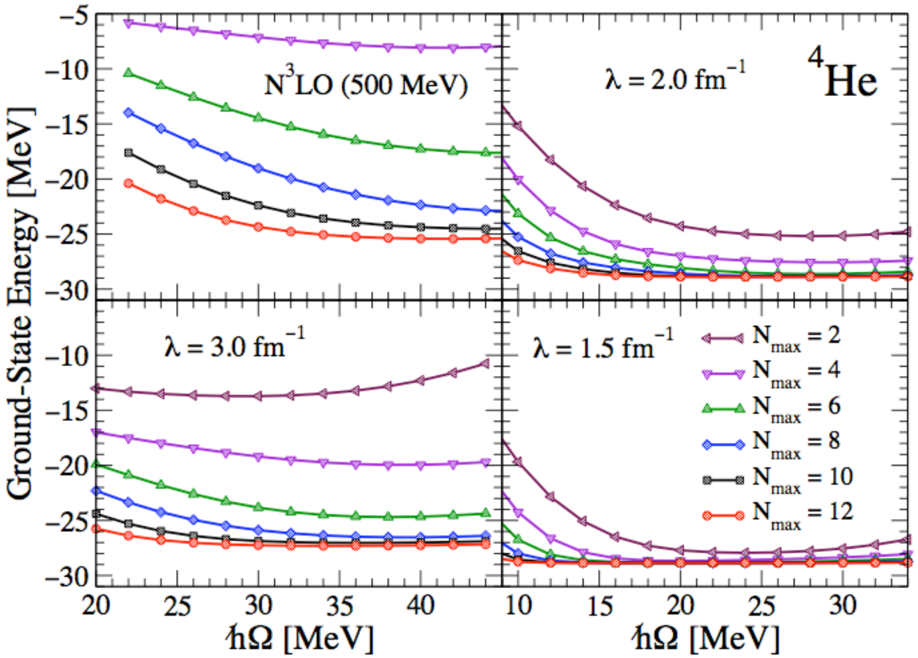


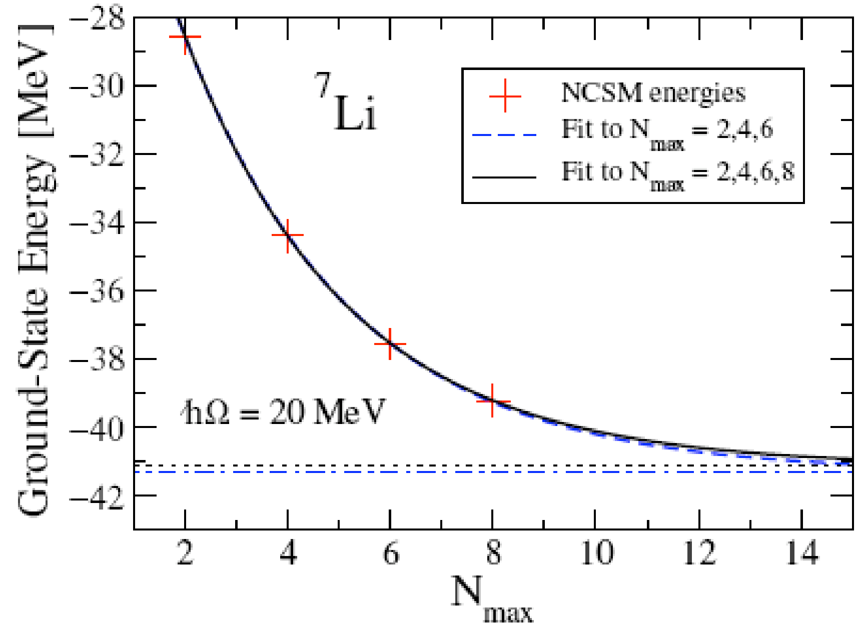
Correlation energy becomes SMALL

Most of 3N/4N effects can be treated in mean field approx.

- Induced NNN (and 4N) grows w/density…BUT

Low momentum interactions in light nuclei





[arXiv:0708.3754]

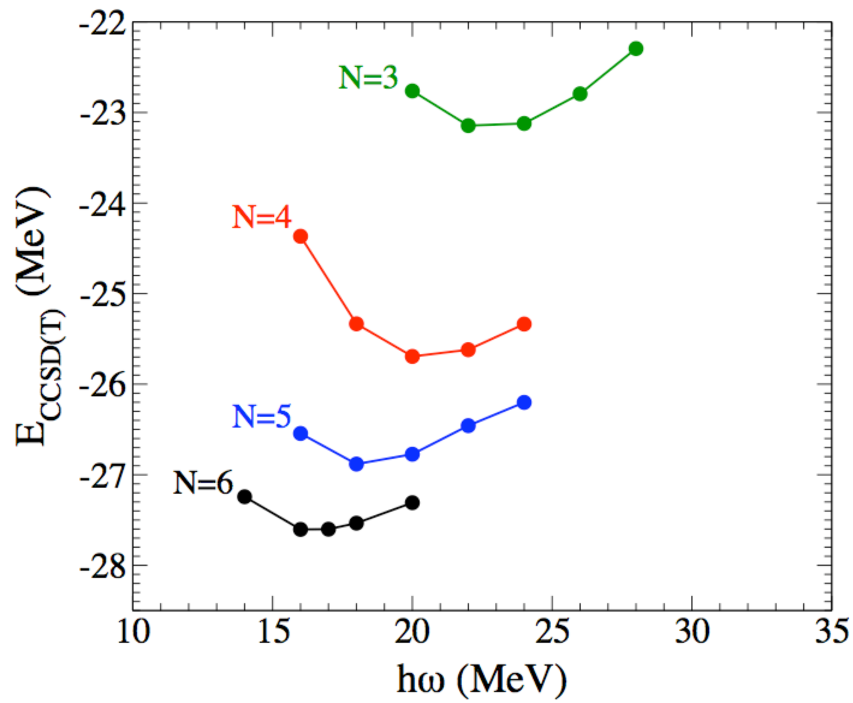
- Rapid convergence in NCSM calculations (NN only so far, $A \leq 7$)
- 1st ever benchmark for O16 (NCSM/Coupled Cluster, in progress)
- Variational => robust exponential extrapolations, weak correlations
- Next up: include 3NF (fit N2LO 3NF) and vary Λ
- Eventually: include evolved 3NF and vary Λ

} Missing 4N, higher order 3N, scaling w/heavier A...

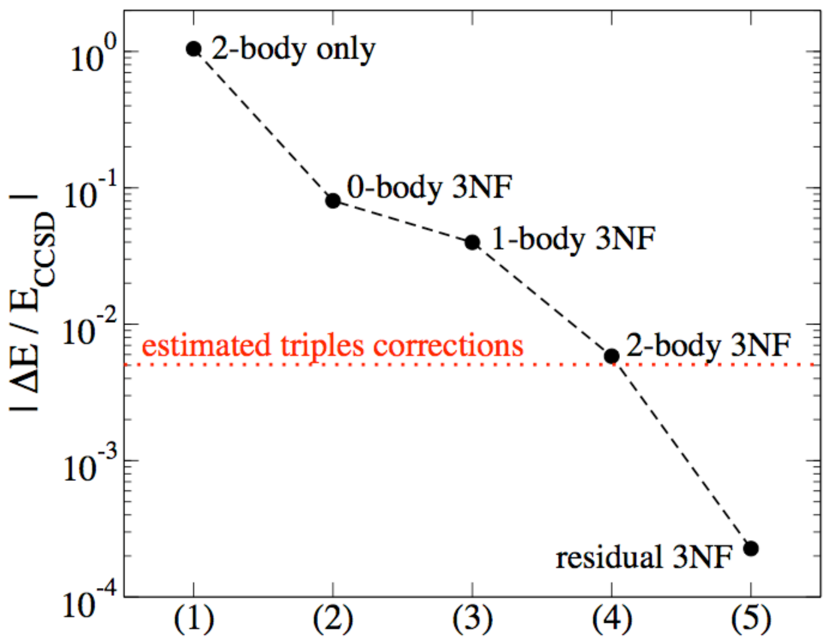
Coupled-Cluster calculations of ^{4}He using $V_{low\ k}$ + NNN

- First **ever** inclusion of NNN (fit N2LO 3NF)
- fast convergence (agrees w/exact FY)
- Gentle scaling to heavier systems

≈ Exact calculations for Ca40 (at present) become possible!



Hagen et.al., arXiv:0704.2854

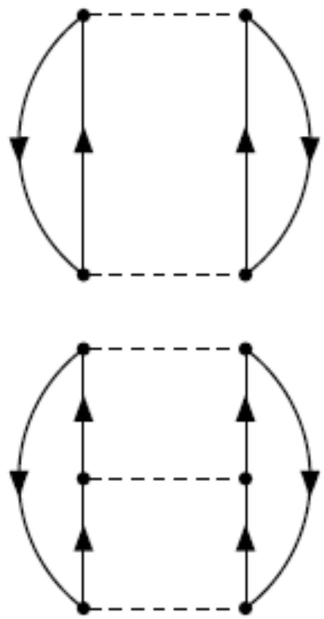


99.9% of 3NF contributions are effectively ρ-dependent 0, 1, 2-body pieces in the normal-ordered Hamiltonian!

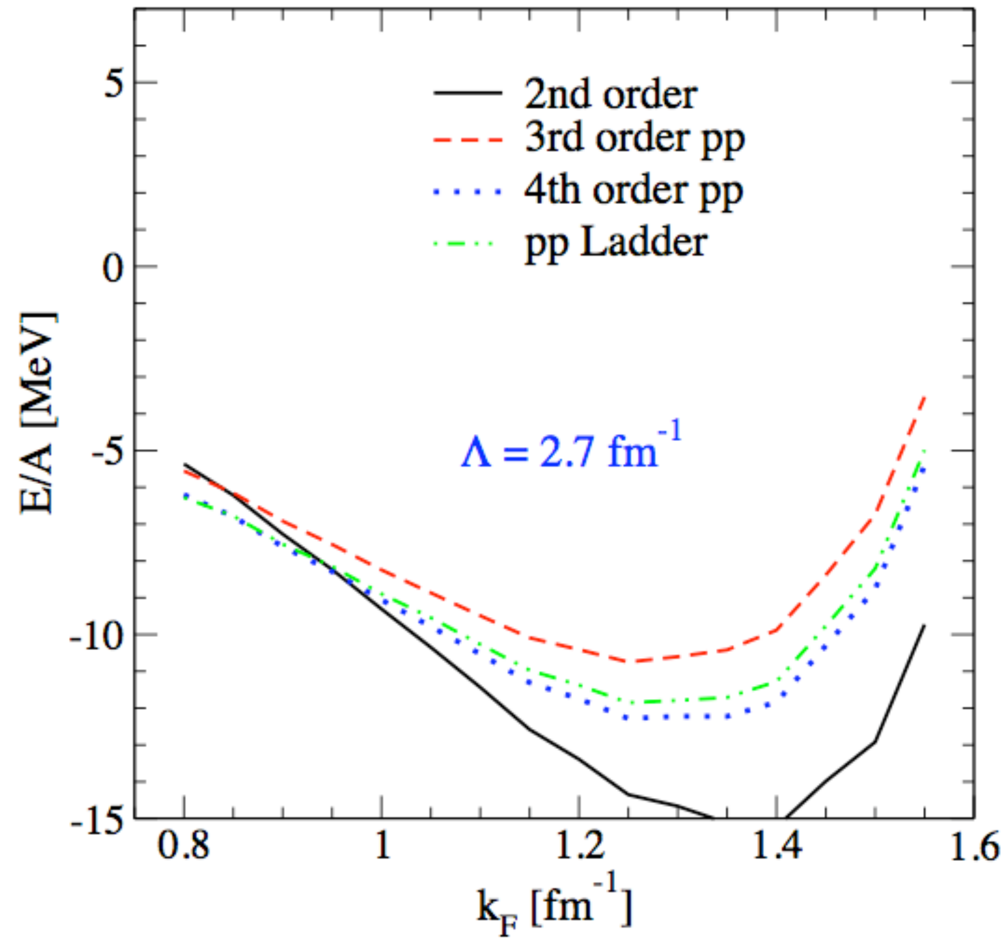
(at least for doubly magic nuclei)

Lowering Λ increases "perturbativeness" in nuclear matter

- Brueckner ladders order-by-order



- 3-body approximated (density-dependent NN)



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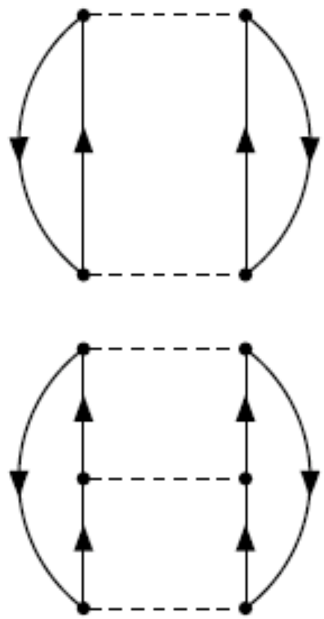

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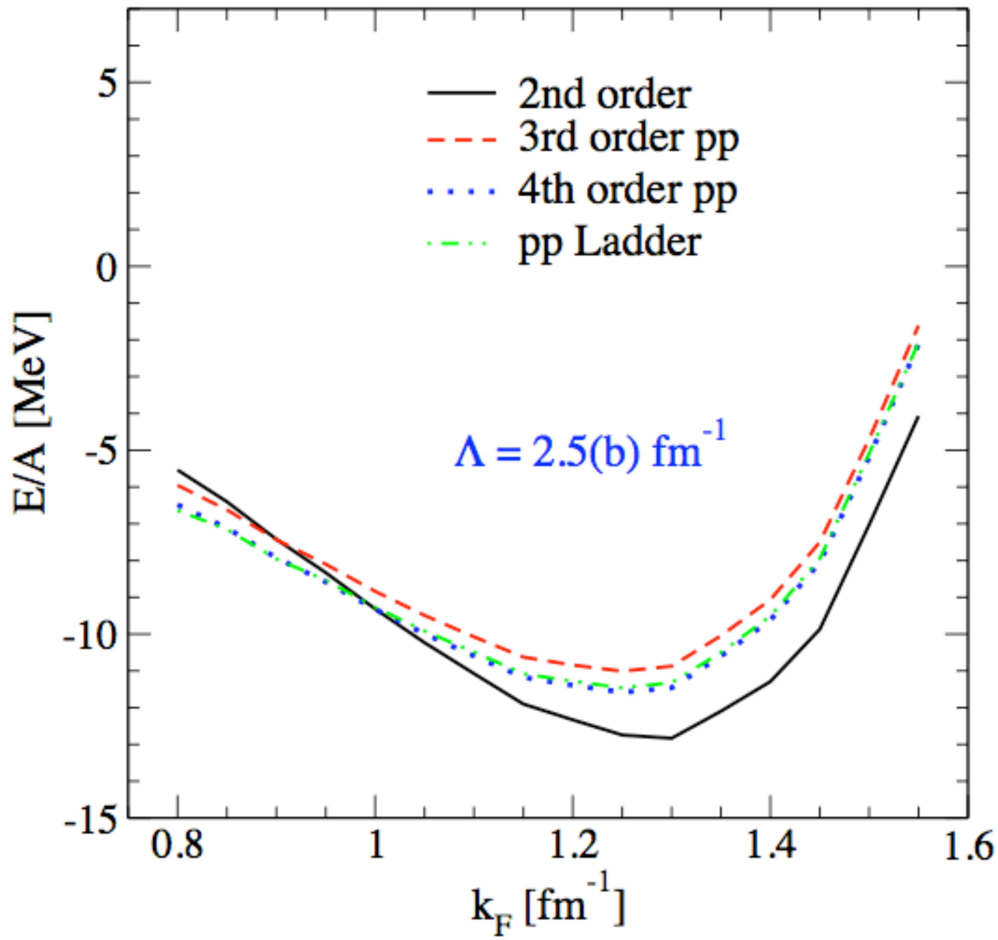

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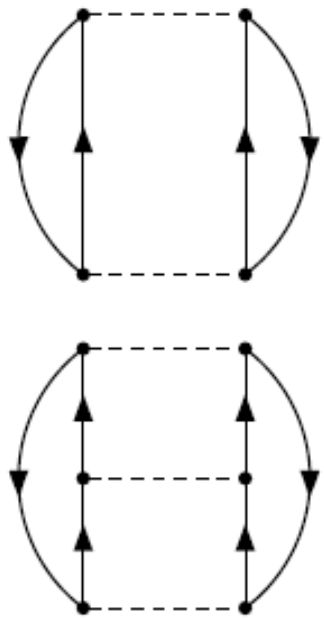

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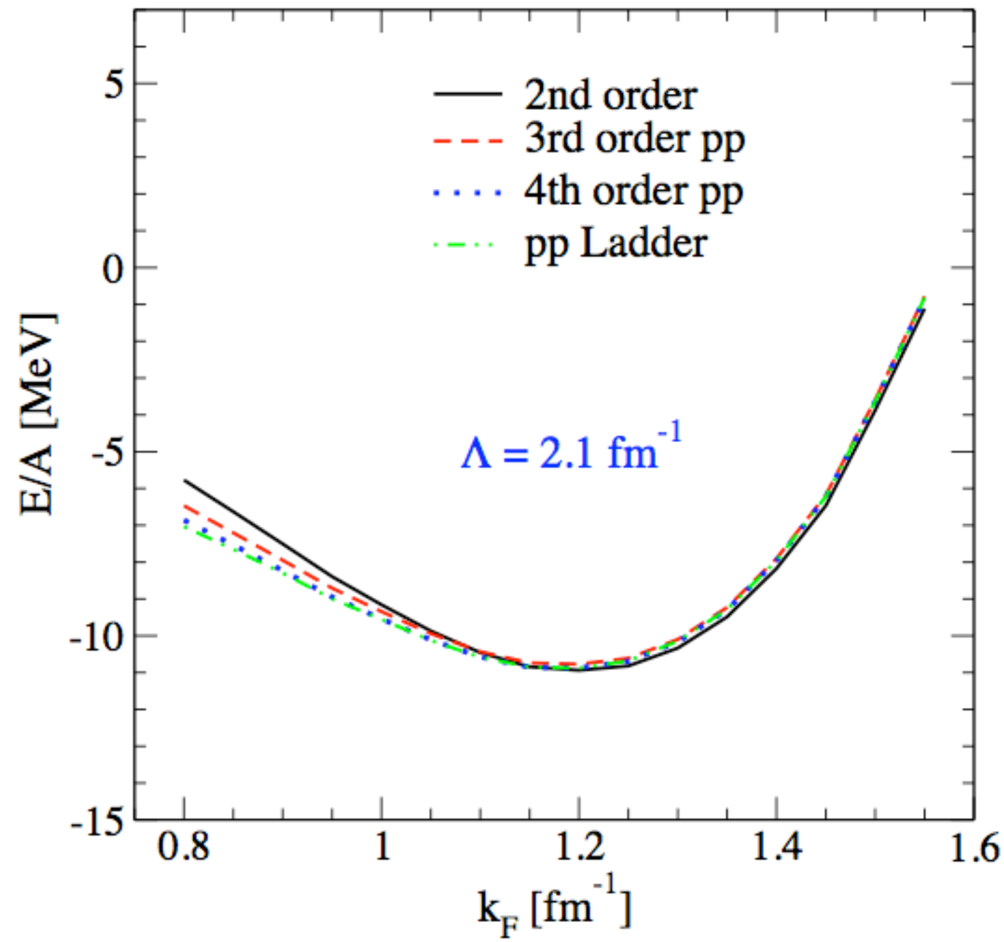

Microscopic density functional theory with Theoretical error bars now possible?
See SciDac project www.unedf.org for details

Conclusions

- Lowering Λ greatly simplifies few and many-body problems
  - -strong short range correlations are blurred out
  - -simple w.f.'s much more effective for variational calcs.
  - -smaller basis sets (shell model, coupled-cluster)
  - -use Λ-dependence as a tool for estimating theoretical errors
  - -3N/4N grow in size, BUT can be treated as perturbation
  - -With Coupled Cluster techniques, quasi-exact calculations of Ca40 are now possible
  - -perturbative nuclear matter => microscopic density functional theory with theoretical error bars
  - -codes now available

- RG machinery exists to evolve NNN and currents
  - -implementation is underway (R. Furnstahl's talk [arXiv:0708.1654])