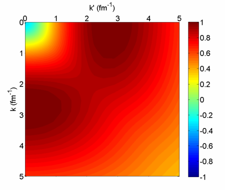
Operator Evolution Using SRG Flow Equations for Few-Body Systems



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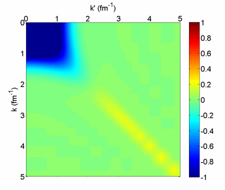

In Collaboration with: S.K Bogner, R.J. Furnstahl, E.D. Jurgenson, & R.J. Perry

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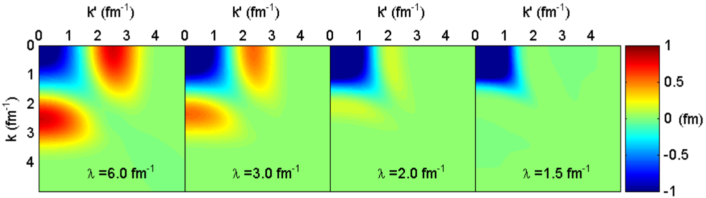
SRG: A Brief Review

- **The Similarity Renormalization Group (SRG)**
  → provides a means to systematically evolve computationally difficult **potentials and operators**

$$O_s = U_s O_{s=0} U_s^\dagger \iff \frac{dO_s}{ds} = [\eta_s, O_s] = [[T_{rel}, H_s], O_s]$$

- Hamiltonian Operator → driven toward diagonal or decoupled form



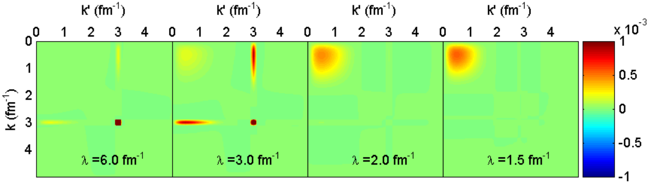
$^3S_1$ **N3LO 500MeV – Entem & Machleidt**

Note:
$\lambda = \frac{1}{s^{1/4}}\ \mathrm{fm}^{-1}$

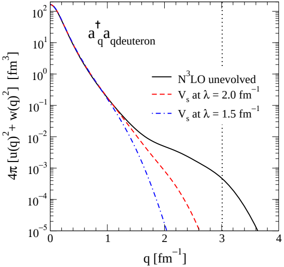
- Unitarily evolve operators consistent with any initial potential
  → Number Operator, Electromagnetic Form Factors, etc.
  → Deuteron and a 1D system of bosons provide "lab"
- Issues: – Decoupling
  – Induced few-body operators
  – Convergence in harmonic oscillator basis

High and Low Momentum operators in the Deuteron

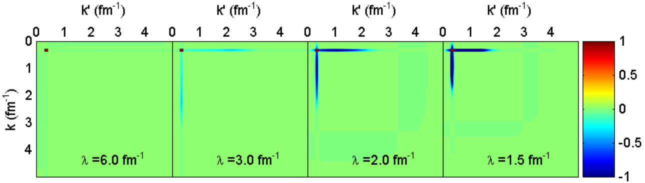
- **Integrand** of $\langle \psi_d | U^\dagger (U a_q^\dagger a_q U^\dagger) U | \psi_d \rangle$ for $q = 3.02\,\mathrm{fm}^{-1}$



- Momentum Distribution



- **Integrand** for $q = 0.34\,\mathrm{fm}^{-1}$



  - High momentum components suppressed
  - Integrated value does not change, but nature of operator does
  - Similar for other long distance operators: $\langle r^2 \rangle$, $\langle Q_d \rangle$, & $\langle \frac{1}{r} \rangle$

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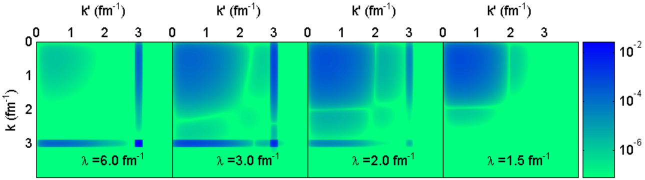

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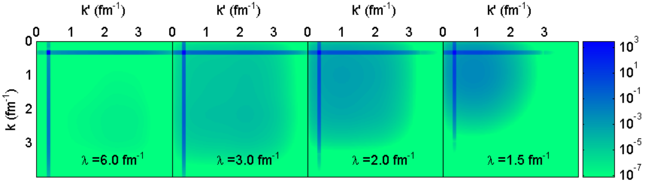

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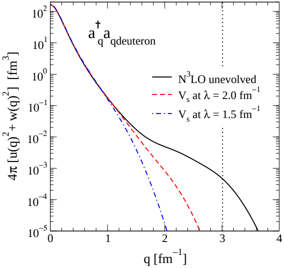

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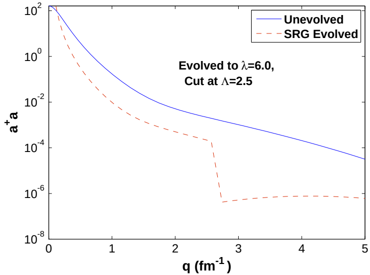
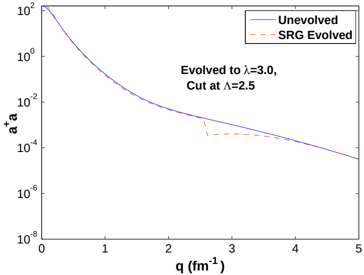
Demonstration of Decoupling In Expectation Values

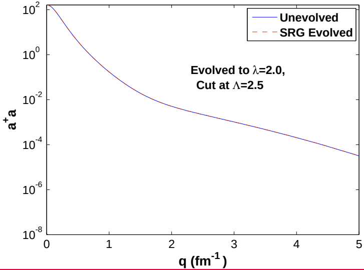
- **Evolve** Hamiltonian & operators **to** λ **in full space** $\rightarrow$ **TRUNCATE at** Λ:





- Momentum distribution
- $\Lambda = 2.5\,\mathrm{fm}^{-1}$

  $\lambda = 6.0\,\mathrm{fm}^{-1}, 3.0\,\mathrm{fm}^{-1}$, and $2.0\,\mathrm{fm}^{-1}$
- Decoupling **for all q** is successful when $\lambda < \Lambda$
- Calculated with AV18 potential.



Many-Body evolution of Operators

- Analysis of many-body evolution with operators normal ordered in the vacuum:

$$\frac{d\widehat{O}_s}{ds} = [[T_{\rm rel}, H_s], \widehat{O}_s] \Longrightarrow \left[\left[\sum_{ij} T_{ij} a_i^\dagger a_j, \sum_{i'j'} T_{i'j'} a_{i'}^\dagger a_{j'} + \frac{1}{2}\sum_{pqkl} V_{pqkl|s} a_p^\dagger a_q^\dagger a_l a_k\right], \widehat{O}_s\right]$$

$\rightarrow$ Only one non-vanishing contraction in the vacuum: $\overbrace{a_i a_j^\dagger} = \delta_{ij}$

- A general operator $\widehat{O}$ for an A-body system can be written as

$$\widehat{O} = \widehat{O}^{(1)} + \widehat{O}^{(2)} + \widehat{O}^{(3)} + \cdots + \widehat{O}^{(A)}$$

where the $\widehat{O}^{(i)}$ label the $i = 1, 2, 3, ..., A$-body components of the operator.
– SRG operator $\widehat{O}_s$ will have contributions at all levels of n so that $\widehat{O}^{(n)} \neq \widehat{O}_s^{(n)}$.

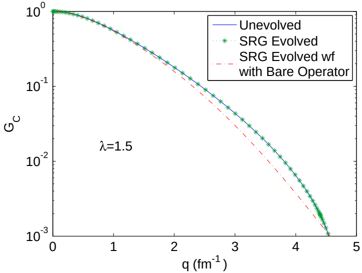
- Expanding commutators and making contractions, one finds:

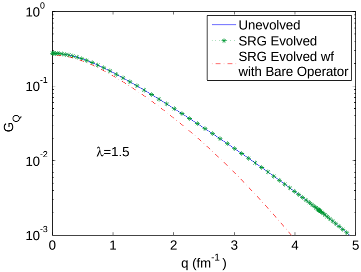
$\rightarrow$ Evolution of an operator is fixed in each n-body subspace
$\rightarrow$ E.g.,

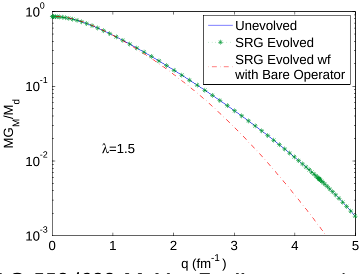
  * Evolve $\widehat{O}$ in a 1-body space to $\widehat{O}_s \Longrightarrow$ determines $\widehat{O}_s^{(1)}$ for all A
    – 1-body operators unchanged with this generator
  * Evolve $\widehat{O}$ in a 2-body space to $\widehat{O}_s \Longrightarrow$ determines $\widehat{O}_s^{(2)}$ for all A
  * Evolve $\widehat{O}$ in a 3-body space to $\widehat{O}_s' \Longrightarrow$ determines $\widehat{O}_s^{(3)}$ for all A, etc.

- **Elastic electron-deuteron scattering**
- Calculation of G_C, G_Q, G_M
- 1-body versus 2-body evolution?

  $\Longleftrightarrow$ bare versus evolved







- Wave function is derived from the **NNLO 550/600 MeV – Epelbaum** *et al.* potential and the evolution is run to $\lambda = 1.5\,\mathbf{fm}^{-1}$.

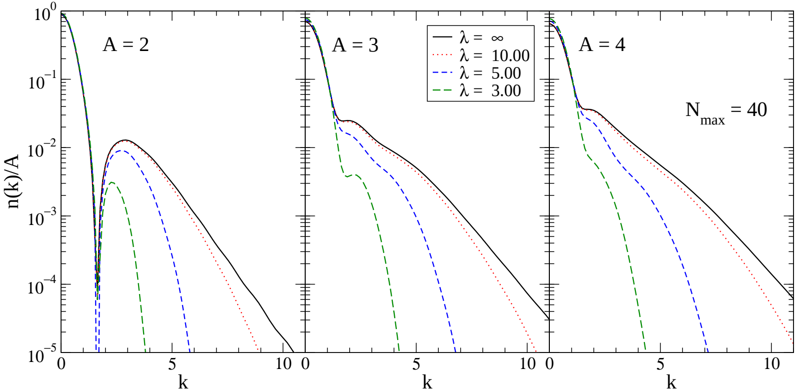
Momentum Distribution in Few-Body Model Space

- **A Test Bed for 3D NCSM calculations:**
- **Embed in 1D few-body HO space** (code from E.D. Jurgenson)

  → System of A bosons interacting via a model potential

  → Here we have: $\langle N_A i_A | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | N_A i_A \rangle$

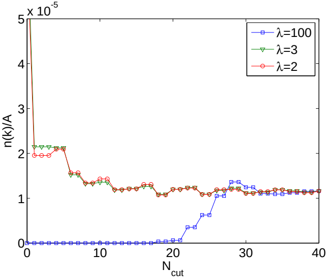
  N_A = Symmetric Jacobi states, i_A =degeneracies

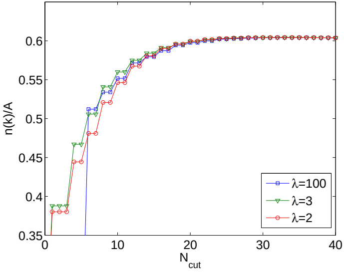


- High momentum components suppressed

High and Low Momentum Operators in Oscillator Basis

- Number operator at: $q = 15.0$ & $q = 0.5$



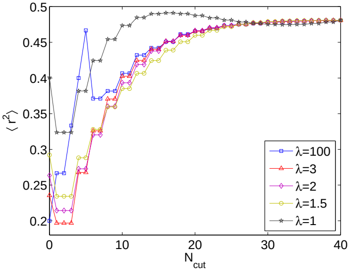


- A=3 System
- **Evolve** Hamiltonian & operators **to** λ **at large** $N_{\max}$
  $\rightarrow$**Truncate** model space at $N_{\rm cut}$
  $\longrightarrow$ Poor convergence of long range operators

$$\Lambda_{UV} \sim \sqrt{mN_{max}\hbar\omega}; \quad \Lambda_{IR} \sim \sqrt{\frac{m\hbar\omega}{N_{max}}}$$

$\longrightarrow$Can this be corrected?

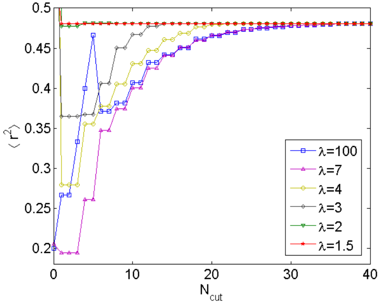
- $\langle r^2 \rangle$ Value



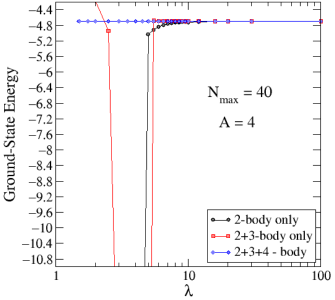
An alternative generator - diagonal in the basis

- The harmonic oscillator Hamiltonian H_{ho} is diagonal in the basis
- Consider

$$\eta_s = [H_{\mathrm{ho}}, H_s]$$



$\langle r^2 \rangle$ for A=3



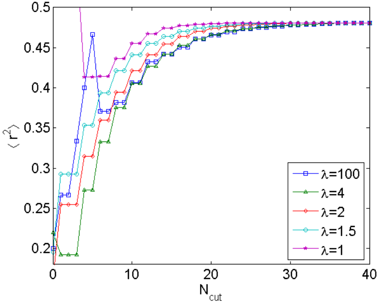
Many Contributions to Binding Energy

- Greatly Improved convergence
- **Generates spurious deep bound states . . .**

Controlled IR and UV renormalization I

- Consider $T_{\rm rel} + \alpha r^2$, where α is a parameter which can be adjusted to optimize the renormalization (here, $\alpha = 1$), so that

$$\eta_s = [T_{\rm rel} + \alpha r^2, H_s]$$



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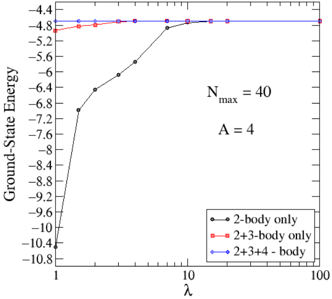
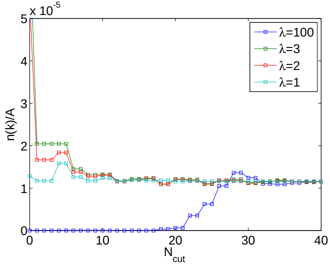
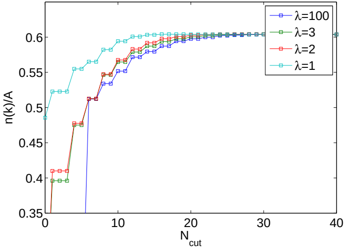


Many Contributions to Binding Energy

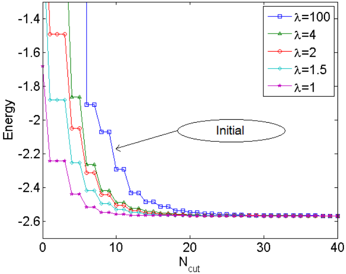
- Convergence improves with decreasing λ
- **No spurious deep bound states**. Is hierarchy of many-body forces under control?

- Convergence of High (q=15.0) and Low (q=0.5) number operators with $\eta_s = [T_{\rm rel} + \alpha r^2, H_s]$





- Convergence of binding energy →



Conclusion/Outlook

Summary:

- Nuclear operators can be consistently evolved and calculated with the SRG
- Many-body contributions can be extracted at each order
- HO Basis is a pathway to few body operators & electromagnetic interactions
- Alternative generators in 1D NCSM

Outlook:

- Variational few-body calculations
- Study alternative generators in 3D NCSM
- Higher order electromagnetic interactions in few-body nuclei

The End