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# Exercises #3

•**Problem 1.** Plot the first Bessel function  $J_0(x)$  in the interval  $\{0, 10\}$ . Find the location of the zeros.

•**Problem 2.** The Legendre polynomials (which arise in quantum mechanics en route to the spherical harmonics) satisfy the relation

$$(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x),$$

with  $P_0(x) = 1$  and  $P_1(x) = x$ . Write a function to generate  $P_n(x)$  recursively. (You can check your function against the built-in `LegendreP` [.] .)

•**Problem 3.** *Projectile Motion.* Consider the motion of a projectile in the  $(x, z)$  plane, neglecting air resistance. The equations of motion are

$$\begin{aligned}mz''(t) &= -mg \\mx''(t) &= 0\end{aligned}$$

If the projectile is fired with initial velocity  $v_0$  at an angle  $\theta$  with respect to horizontal, find the range, i.e. the  $x$ -coordinate when it hits the ground again. Verify that for any velocity the range is maximized for  $\theta = \frac{\pi}{4}$ .

•**Problem 4.** *Plus air resistance.* Now add in air resistance, which we model as a force depending linearly on the velocity. The modified equations are

$$\begin{aligned}mz''(t) &= -mg - bz'(t) \\mx''(t) &= -bx'(t)\end{aligned}$$

Again solve for the range. Taking for definiteness the values  $g = 9.8m/s^2$ ,  $m = 1\text{kg}$ ,  $b = .2N \cdot s/m$ , what is the optimal angle for  $v_0 = 10\text{m/s}$ ? For  $v_0 = 100\text{m/s}$ ? Make a plot of the angle as a function of  $v_0$ . If you still have time, consider some fixed angle, say  $\frac{\pi}{4}$ , and make a plot of the final velocity of the projectile as a function of the initial velocity. Explain the limiting behavior of this function for small and for large initial velocities.