
Exercises #2

•**Problem 1.** Compute $\sqrt{16}$ using three different syntaxes: `Sqrt[16]`, `16 // Sqrt` and `Sqrt @ 16`. Predict the result for `Sin @ 16 // Sqrt`.

•**Problem 2.** Understand the output for the following:

```
In[1]:= Tan @ Sqrt[x] /. x-> 16
Sqrt[x] // Tan /. x->16
(x^2 + y^2) /. {x->r Sin[a], r->1}
(x^2 + y^2) //. {x->r Sin[a], r->1}
(x^2 + Sin[y]) /. {x->y, y->x}
(x^2 + Sin[y]) //. {x->y, y->x}
```

•**Problem 3.** Last time you solved the following set of equations, presumably using the `Solve[]` function.

$$2I_1 + 3I_2 - 4I_3 = 5$$

$$3I_1 - 2I_2 + 4I_3 = 6$$

$$4I_1 + 1I_2 - 4I_3 = 7$$

Solve the same problem by computing the inverse of the matrix of resistances defined by

```
In[2]:= m = {{2,3,-4}, {3,-2,4}, {4,1,-4}}
```

and applying the inverse to the vector of voltages

```
In[3]:= v = {5,6,7}
```

•**Problem 4.** Find the eigenvalues of the matrix defined above. Do this two ways: (a) using `Eigenvalues`, and (b) by constructing the characteristic polynomial

```
In[4]:= Det[ m - lambda*{{1,0,0},{0,1,0},{0,0,1}} ]
```

and solving for its roots.

•**Problem 5.** In single slit diffraction we encounter the function $\sin^2(x)/x^2$, which has a maximum at $x = 0$, and smaller peaks at points near $x \sim (n + \frac{1}{2})\pi$. Find the precise location of the $n = \frac{3}{2}$ and $n = \frac{5}{2}$ peaks. What percentage error do we make in saying “ $x = (n + \frac{1}{2})\pi$ ”?

•**Problem 6.** Expand $\sin^2(x)/x^2$ in an 8th order Taylor series and find the first two zeroes along the real axis. How close are they to π and 2π ? Repeat for 16th and 24th order.

•**Problem 7.** Try a few indefinite integrals, e.g.

$$\int \frac{1}{a^2 + x^2} dx$$

$$\int \frac{\log(x)}{x} dx$$

$$\int \frac{\sin(x)}{\sqrt{x}} dx$$

In each case verify the answer by applying D to the result.

•**Problem 8.** A few definite integrals:

$$\int_0^1 \sqrt{1-x^2} dx$$

$$\int_0^1 \frac{1}{\sin^2(\sqrt{\pi^2+x})} dx$$

$$\int_0^\infty \exp(-x^2)x^4 dx$$

Try that last one with an arbitrary parameter n :

$$\int_0^\infty \exp(-x^2)x^n dx$$

An impressive one:

$$\int_0^1 \frac{\log(x)^4}{\sqrt{1-x^2}} dx$$

The volume of sphere:

$$8 \int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \sqrt{1-x^2-y^2}$$

•**Problem 9.** Here is an integral Mathematica can't do in closed form:

$$f(\lambda) \equiv \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-x^2 - \lambda x^4) dx$$

If λ is small, we can get a good series approximation for $f(\lambda)$. Find this approximation by expanding the integrand in powers of λ and integrating term by term.

For definiteness take $\lambda = .025$ and make a table of results when keeping n terms, where n ranges up to 12. (You can generate even more terms if you have patience.) Compare against the “exact” result from the numerical integral. The results should look pretty good.

Now repeat for $\lambda = .05$. It looks like the series not converging. What is going on?