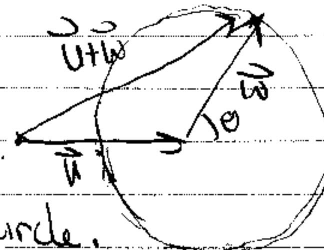
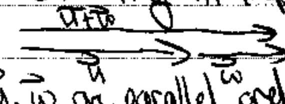
(Two acceptable solutions here!) ②

Example: [C2S.2]

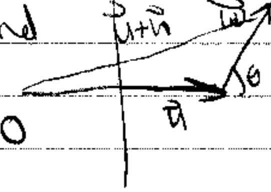


Use a diagram for general $\vec{u}, \vec{w}$.
If we imagine rotating $\vec{w}$ with all possible θ, the tip will be on the circle.
The circle gets furthest from the beginning of $\vec{u}$, maximizing mag$(\vec{u}+\vec{w})$, when $\vec{u}$ and $\vec{w}$ are parallel $\Rightarrow \theta = 0$. Only in this case will mag$(\vec{u}+\vec{w})$ = mag$(\vec{u})$ + mag$(\vec{w})$



So mag$(\vec{u}+\vec{w})$ = mag$(\vec{u})$ + mag$(\vec{w})$ when $\vec{u}, \vec{w}$ are parallel and pointing in the same direction and mag$(\vec{u}+\vec{w})$ is always less than or equal to mag$(\vec{u})$+ mag$(\vec{w})$. To get zero, we must have them equal and opposite $\Rightarrow \boxed{\vec{u} = -\vec{w}}$

Alternative solution with components:



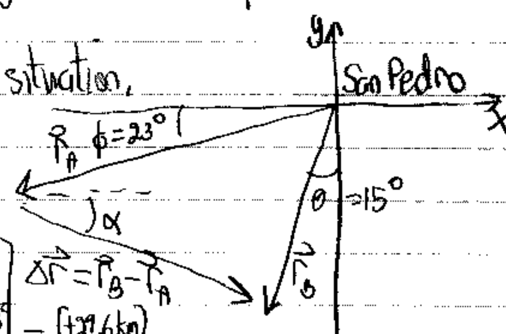
Choose $\vec{u}$ along the x-axis, $\vec{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}$ and
$\vec{w}$ at angle θ: $\vec{w} = \begin{bmatrix} w_x \\ w_y \end{bmatrix} = \begin{bmatrix} w\cos\theta \\ w\sin\theta \end{bmatrix}$ where $w = \text{mag}(\vec{w}) > 0$

Then the magnitude of $\vec{u}+\vec{w}$ is

$$\text{mag}(\vec{u}+\vec{w}) = \sqrt{(u_x+w_x)^2+(u_y+w_y)^2} = \sqrt{(u+w\cos\theta)^2+(0+w\sin\theta)^2}$$
$$= \sqrt{u^2+2uw\cos\theta+w^2(\cos^2\theta+\sin^2\theta)} = \sqrt{u^2+2uw\cos\theta+w^2}$$

But mag$(\vec{u}) = u$ and mag$(\vec{w}) = w$, so only if $\cos\theta = 1$ does mag$(\vec{u}+\vec{w})$ = mag$(\vec{u})$ + mag$(\vec{w})$. Otherwise $u^2+2uw\cos\theta+w^2 < (u+w)^2$.
For mag$(\vec{u}+\vec{w}) = 0$, $u = w$ and $\cos\theta = -1$, or $\vec{u} = -\vec{w}$.
Since $-1 \le \cos\theta \le 1$, mag$(\vec{u}+\vec{w})$ is always less than or equal to $u+w$.

Example: [C2S.6] The diagram shows the situation.



mag$(\vec{r}_A) = 42$km and mag$(\vec{r}_B) = 35$km.
We want the magnitude and angle of $\Delta\vec{r}$.
Use components:

$$\Delta\vec{r} = \begin{bmatrix} -r_B\sin\theta \\ -r_B\cos\theta \\ 0 \end{bmatrix} - \begin{bmatrix} -r_A\cos\phi \\ -r_A\sin\phi \\ 0 \end{bmatrix} = \begin{bmatrix} -(35\text{km})\sin 15° \\ -(35\text{km})\cos 15° \\ 0 \end{bmatrix} - \begin{bmatrix} -(42\text{km})\cos 23° \\ -(42\text{km})\sin 23° \\ 0 \end{bmatrix} = \begin{bmatrix} +29.6\text{km} \\ -17.4\text{k} \\ 0 \end{bmatrix}$$

Then mag $\Delta\vec{r} = \sqrt{(29.6\text{km})^2+(-17.4\text{km})^2+0^2} = \boxed{34.3\text{km}}$ and $\alpha = \tan^{-1}\left|\frac{\Delta y}{\Delta x}\right| = \tan^{-1}\left|\frac{-17.4\text{km}}{29.6\text{km}}\right| = \boxed{30.4°}$

Sample Answers to H131 Homework

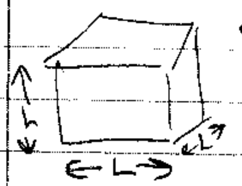
- **Two-minute questions**: Your answer can be one or two sentences only. This is not easy to do well!
  Example: [C2T.7] False: The magnitude $mag(\vec{u}) = \sqrt{u_x^2 + u_y^2 + u_z^2}$ is the positive square root. Note that u_x^2, u_y^2, u_z^2 are always ≥ 0 even if u_x, u_y, or u_z is negative.

- **Basic Problems**: These do not require detailed explanations, but you must include intermediate steps.
  Example: [C2B.10] Find $\vec{u} + 2\vec{w}$ by multiplying and adding components:

$$\vec{u} + 2\vec{w} = \begin{bmatrix} u_x + 2w_x \\ u_y + 2w_y \\ u_z + 2w_z \end{bmatrix} = \begin{bmatrix} 2m + 2(-4m) \\ -3m + 2(-1m) \\ 1m + 2(3m) \end{bmatrix} = \begin{bmatrix} -6\,m \\ -5\,m \\ +7\,m \end{bmatrix}$$

- **Synthetic, Rich Context, and Advanced Problems**
  - Your solutions should have words explaining your strategy and important intermediate steps. You should draw and label appropriate diagrams. State your assumptions.

Example: [C1S.5] Let the rock be length L on a side and total mass m_{rock}. Take mass $m_{person} \simeq 60\,kg$ from Figure C1.8c



- Given density $\rho_{rock} \approx 3\rho_{H_2O} \approx 3\rho_{person}$, assuming that a person is mostly water.
- Density is mass per volume, so $m = \rho * V$.

So $m_{rock} > m_{person} \Rightarrow \rho_{rock} V_{rock} > m_{person}$ or $V_{rock} > \frac{60\,kg}{3000\,kg/m^3} = 0.02\,m^3$
The rock is cubic so $L = \sqrt[3]{V_{rock}} > \sqrt[3]{0.02\,m^3} = 0.27\,m\left(\frac{100\,cm}{1\,m}\right) = \boxed{27\,cm}$
So the rock should be bigger than about 27 cm on a side.
[bigger or smaller for different size people!]

put a box around any final answers