Review Sheet for H131 Midterm Exam

What you should know about . . .

Units and Vectors and Significant Figures [C1,C2,C8.2,C7.6]

1. Conversion of units [C1.9] and basic SI units of length (meter or m), time (second or s), mass (kilogram or kg), force (Newton or N), energy or work (Joule or J), power (Watt or W). Angles in radians (and $1\,\text{rev} = 2\pi\,\text{radians}$).

2. Basic vector representations and operations

    - Scalars versus vectors; recognizing common errors [C3.6]
    - Addition, subtraction, multiplication/division by scalar both pictorially and with components
    - Magnitude $u = \text{mag}(\vec{u}) = |\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

3. Dot products $\vec{a} \cdot \vec{b} = ab\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$, or $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ [C8.2]

    - $\vec{a} \cdot \vec{b} = 0$ when $\vec{a} \perp \vec{b}$ (or one is zero)
    - $|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$

4. Basic vector relations like $|\vec{p} + \vec{q}| \leq |\vec{p}| + |\vec{q}|$

5. Know how many significant figures to use in your answers

What you should know about . . .

Momentum and Impulse [C3,C4,C5]

1. A particle's momentum is $\vec{p} = m\vec{v}$ where $\vec{v} = d\vec{r}/dt$ (so $d\vec{r} = \vec{v}\,dt$ for short enough dt)

    - Speed $v = |\vec{v}|$ is a (positive) scalar; motion in circle has constant speed but not constant momentum

2. Total (or net) impulse is vector sum of those from interactions: $d\vec{p} = [d\vec{p}]_A + [d\vec{p}]_B + \cdots$ and these may add to zero for one or more components

3. Force from interaction A defined as impulse $[d\vec{p}]_A$ during time dt, or $\vec{F}_a \equiv [d\vec{p}]_A/dt$

4. Mass measures how much impulse is needed to change a particle's velocity while weight $\vec{F}_g$ is the force on the particle from gravitaional interactions.

    - $\vec{F}_g = m\vec{g}$, where $\vec{g}$ is the gravitational field vector

- Near the earth's surface, $\vec{g}$ points toward the earth's center with magnitude $g = 9.8\,\mathrm{N/kg}$.

5. Center of mass $\vec{r}_{\mathrm{CM}} \equiv \frac{1}{M}(m_1\vec{r}_1 + \cdots + m_N\vec{r}_N)$
   - Total momentum and center of mass velocity related by $\vec{p}_{\mathrm{tot}} = M\vec{v}_{\mathrm{CM}}$
   - Particle model: System's center of mass responds to external interactions like a point particle
6. Inertial reference frames are where Newton's first law holds. In practice, a frame attached to the center of mass of a system is inertial if the frame is not rotating and the system's only external interactions are far away gravitational interactions, which are ignored.
7. Momentum conservation applies if a system floats in space or is functionally isolated (net flow from external interactions is zero) or is momentarily isolated (brief but strong internal interaction; e.g., collision).
   - Know how to solve basic conservation of momentum problems, with before and after pictures, reference frame axes, labels, and known values.

What you should know about . . .

Potential Energy [C6, C7, C11]

1. Total energy $E = K_1 + K_2 + \cdots + V(r_{12}) + V(r_{13}) + V(r_{23}) + \cdots$
2. Reference separation is r_0 such that $V(r_0) = 0$
3. Examples of potential energy:
   - Near earth's surface, $V(z) = mgz$
   - Gravitational interaction (reference separation $r_0 \to \infty$) $V(r) = -G\frac{m_1 m_2}{r}$
   - Electromagentic interaction (reference separation $r_0 \to \infty$) $V(r) = +k\frac{q_1 q_2}{r}$
   - Spring $V(r) = \frac{1}{2}k_s(r - r_0)^2$ with "spring constant" k_s
4. Potential energy diagrams, including forbidden and allowed regions, turning points, and equilibrium positions

What you should know about . . .

Kinetic Energy and k-Work [C8,C9]

1. Kinetic energy $K = \frac{1}{2}mv^2 = p^2/(2m)$; change in kinetic energy due to change in momentum $dK = \vec{v} \cdot d\vec{p} = v\,dp\,\cos\theta$ [C8]

2. Forces generate energy and momentum transfers: $d\vec{p} = \vec{F}\,dt$; $dK = \vec{F} \cdot d\vec{r}$ [C8]

   - k-Work $[dK]_A = \vec{F}_A \cdot d\vec{r}$ by interaction A can be positive, negative, or zero

3. Rotational kinetic energy $K^{\text{rot}} = \frac{1}{2} I \omega^2$ [C9]

   - Angular speed $\omega = |d\theta/dt|$ (SI units: rad/s)
   - Ordinary speed along a circular path of radius r is $v = r\,\omega$
   - Direction of angular velocity $\vec{\omega}$ from right hand rule
   - Moment of inertia $I = \sum_{i=1}^{N} m_i r_i^2$ where m_i is the mass of the ith particle and r_i is its distance from the axis of rotation
   - An object's I depends on its mass, shape, and the axis of rotation

4. Separating translation and rotation:
   Total kinetic energy of a moving and rotating macroscopic object:
   $K = K^{\text{cm}} + K^{\text{rot}} = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I \omega^2$.

5. Rolling without slipping: speed of center of mass and rotational speed are related by $\omega = v_{\text{cm}}/R$ where R is radius of rolling object.

   - This implies that if $I = \mu M R^2$ (μ depends on mass distribution) then $K^{\text{rot}} = \frac{1}{2} \mu M v_{\text{cm}}^2 = \mu K^{\text{cm}}$ and $K = (1 + \mu) K^{\text{cm}}$.
   - This explains why rolling objects are slower on an incline than objects of the same mass that have a smaller moment of inertia or that slide without friction.

What you should know about ...

Other Energy Forms and Energy Conservation [C10-C12]

1. Thermal energy U^{th} is a special form of internal energy U

   - An object's temperature T is proportional to its molecules' average kinetic energy
   - In addition to molecular kinetic energy, thermal energy contains molecular vibrational and rotational energy and potential energy between molecules
   - This means that in general U^{th} is a complicated function of T
   - In a limited temperature interval (which does not include a phase transition) the change in thermal energy is proportional to change in T: $dU^{\text{th}} = m\,c\,dT$ where m is object's mass and c is its *specific "heat"*.
   - The specific "heat" c is a material constant that depends on the temperature range and changes dramatically near phase transitions

- $T = 0\,\mathrm{K} = -273.15\,°\mathrm{C}$ is absolute zero temperature

2. Heat Q and Work W:
   - Heat Q is energy flow across an objects boundary due to a temperature difference on the two sides of the boundary.
   - Work W is any other energy flow across that boundary
   - Conservation of energy implies $\Delta U = Q + W$
3. Latent "heat" L: Change in thermal energy due to formation or breaking of molecular bonds during a phase transition *that is not accompanied by a change in temperature.* $\Delta U^{\mathrm{la}} = \pm mL$ where the sign depends on the direction of energy flow.
4. Energy conservation: $0 = \Delta K + \Delta V + \Delta U$. Here:
   - $\Delta E = E_f - E_i$ is the change in any one of these energy forms during the process (signs!)
   - $\Delta K = \Delta K^{\mathrm{cm}} + \Delta K^{\mathrm{rot}}$
   - $\Delta U = \Delta U^{\mathrm{th}} + \Delta U^{\mathrm{la}} + \Delta U^{\mathrm{ch}} + \Delta U^{\mathrm{nu}} + \ldots$ is a sum over all internal energy contributions
   - Be able to solve general energy conservation problems (with features as in momentum conservation problems discussed above).
5. Elastic and inelastic collisions:
   - *Elastic:* kinetic energy is conserved
   - *Inelastic:* some kinetic energy is transferred to internal energy
   - *Completely inelastic:* the objects stick together after the collision
6. Power $P = dK/dt$ is the rate at which an interaction transfers energy (SI unit: W = J/s). Power is related to force by $P = \vec{F} \cdot \vec{v}$ where $\vec{v}$ is the velocity of the object on which the force $\vec{F}$ acts.

What you should know about ...

Checking Your Answers [C5.3 and Polya handout]

1. Units (*always, always, always!*)
2. If numerical, is the sign and order of magnitude of your answer reasonable?
3. Check limiting or special cases or trends as you vary a parameter.
4. Try a special case where the answer is easy or known.