

Physics 263: BTM Problem Set #7

Although this is called “BTM Problem Set #7”, none of the problems are actually from BTM, although they are clear extensions. They were selected from standard texts as further practice of an important class of problems: integration over three-dimensional volume elements. Please ask questions! It is due by 5:30pm in the box in 1011 on Friday, April 21.

1. **Rotating cube.** Find the moment of inertia of a uniform cube of mass M with sides of length L about an axis through the centers of two opposite sides.
2. **Solid cone.** Find the mass and moment of inertia about the symmetry axis of a cone of height h and base radius R . The density of the cone varies linearly with the height according to

$$\rho(z) = \rho_0(1 + z/h) ,$$

where $z = 0$ corresponds to the base of the cone. Express your answers in terms of h , R , and ρ_0 .

3. **Charged hemisphere.** A hemisphere of radius R made of an insulating solid has a variable charge density $\rho_{\text{ch}}(\mathbf{r})$. If the hemisphere has its flat side centered on the x - y plane, then $\rho_{\text{ch}}(\mathbf{r})$ is given by

$$\rho_{\text{ch}}(\mathbf{r}) = \rho_0 \frac{r \cos \theta}{R}$$

for $r \leq R$ and zero for $r > R$. Find the total charge in terms of ρ_0 and R .

4. **BONUS: Exponential sphere.** Find the moment of inertia of a sphere of radius R and mass M about an axis through the center if the density (mass per volume) varies as

$$\rho(\mathbf{r}) = \rho_0 e^{-kr/R} .$$

for $r \leq R$ (and is zero for $r > R$). Express your answer in terms of R , M , and k . Show that the limit $k \ll 1$ makes sense.