Name: _____

Physics 263: Chapter 8 Practice Problems I

These are practice for the first part of Chapter 8 of BTM, which covers matrices and determinants.

2×2 Matrix Review

If we have a 2×2 matrix M written as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

then the inverse M^{-1} and the determinant $|M|$ are given by

$$M^{-1} = \frac{1}{|M|} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \qquad \text{where} \qquad |M| = M_{11}M_{22} - M_{12}M_{21} \ .$$

1. The rotation matrix R_θ, which rotates a vector with components (x, y) by angle θ to the vector (x', y'), is given by

   $$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

   Verify that $R_{\theta+\theta'} = R_{\theta'} R_\theta$. What is the determinant of R_θ?

2. Solve the following system of equations by computing a matrix inverse [$x = 3$, $y = -4$]:

   $$\begin{aligned} 3x - 2y &= 17 \\ 5x + 3y &= 3 \end{aligned}$$

3. Why is there not a unique solution to the following equations? What is the value of the appropriate determinant?

   $$\begin{aligned} 6x + 9y &= 3 \\ 2x + 3y &= 1 \end{aligned}$$

Larger Matrices

1. Solve the following simultaneous equations by matrix inversion:

$$\begin{aligned} 2x + 3y - 4z &= -3 \\ 3x - 2y + 5z &= 24 \\ x + 4y - 3z &= -6 \end{aligned}$$

2. Show that

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -5 & 4 \\ -1 & 3 & -2 \end{bmatrix}$$

3. Find the cofactor matrix and the inverse for the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

   Verify that your inverse does what it is supposed to do.

4. Does the following system have a unique solution?

$$\begin{aligned} x + 3y - 2z &= 4 \\ 2x - y + z &= 3 \\ 4x - 9y + 7z &= 1 \end{aligned}$$