Name: ______________________________

Physics 263: Chapter 7 Practice Problems, Part II

These are practice for the second part of Chapter 7 of BTM, which covers curl and divergence, and related topics.

Curl, Path Independence, and Green's Theorem

1. Show that $I \equiv \int_{(1,2)}^{(3,4)} [(6xy^2 - y^3)dx + (6x^2y - 3xy^2)dy]$ is independent of the path joining $(1,2)$ and $(3,4)$ by evaluating the appropriate curl.

2. Evaluate the integral I directly on any path you choose from $(1,2)$ to $(3,4)$.

3. Find the scalar function whose gradient equals the function integrated in I (that is, if $I = \int_{(1,2)}^{(3,4)} \vec{\mathbf{W}} \cdot d\vec{\mathbf{r}}$, find ϕ such that $\boldsymbol{\nabla}\phi = \vec{\mathbf{W}}$).

4. Finally, use the result from the last part to do the integral (using $\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \boldsymbol{\nabla}\phi \cdot d\vec{\mathbf{r}} = \phi(\vec{\mathbf{r}}_f) - \phi(\vec{\mathbf{r}}_i)$).

5. For each of the following vector fields $\vec{\mathbf{W}}$, evaluate $\boldsymbol{\nabla} \times \vec{\mathbf{W}}$. If a scalar field exists for which $\boldsymbol{\nabla}\phi = \vec{\mathbf{W}}$), find ϕ.

   a. $\vec{\mathbf{W}} = y^3\,\hat{\mathbf{i}} - xy^2\,\hat{\mathbf{j}}$

b. $\overrightarrow{\mathbf{W}} = e^{-x}\,\widehat{\mathbf{i}} + \cos y \cos z\,\widehat{\mathbf{j}} - \sin y \sin z\,\widehat{\mathbf{k}}$

c. $[2x \tan y - (\ln z)/x^2]\,\widehat{\mathbf{i}} + x^2 \sec^2 y\,\widehat{\mathbf{j}} + (1/xz)\,\widehat{\mathbf{k}}$

Divergence and Gauss' Theorem

1. For each of the following vector fields $\overrightarrow{\mathbf{W}}$, evaluate $\boldsymbol{\nabla} \cdot \overrightarrow{\mathbf{W}}$.

   a. $\overrightarrow{\mathbf{W}} = \sin^2 x\,\widehat{\mathbf{i}} + y^3\,\widehat{\mathbf{j}} + \cos y\,\widehat{\mathbf{k}}$.

   b. $\overrightarrow{\mathbf{W}} = y^3\,\widehat{\mathbf{i}} - xy^2\,\widehat{\mathbf{j}}$

   c. $\overrightarrow{\mathbf{W}} = e^{-x}\,\widehat{\mathbf{i}} + \cos y \cos z\,\widehat{\mathbf{j}} - \sin y \sin z\,\widehat{\mathbf{k}}$

   d. $[2x \tan y - (\ln z)/x^2]\,\widehat{\mathbf{i}} + x^2 \sec^2 y\,\widehat{\mathbf{j}} + (1/xz)\,\widehat{\mathbf{k}}$

2. Verfiy Gauss' Theorem for the vector $\overrightarrow{\mathbf{W}} = y\widehat{\mathbf{i}} + x\widehat{\mathbf{j}} + z^2\widehat{\mathbf{k}}$ for the cylindrical region bounded by $x^2 + y^2 = R$, $z = 0$, and $z = h$.

3. Show that the integral $\int_S \vec{\mathbf{r}} \cdot d\overrightarrow{\mathbf{S}}$ over *any* closed surface S is equal to three times the volume enclosed by S.